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A
COMPENDIOUS SYSTEM
OF
Natural Philosophy.

With NOTES
Containing the MATHEMATICAL
DEMONSTRATIONS and some
occasional REMARKS.

PART I.
OF
The Properties of BODIES.
Their Laws of MOTION.
And
The Mechanical POWERS.

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COMPENDIOUS SYSTEM

OF

Natural Philosophy.

PART I.

The INTRODUCTION.

SO wild and extravagant have been the Notions of a great part of Philosophers, both ancient and modern, that it is hard to determine, whether they have been more distant in their sentiments from truth, or from one another; or have not exceeded the fancies of the most fabulous Writers, even Poets and Mythologists. This was owing to a precipitate proceeding in their searching into Nature, their neglecting the use of Geometry and Experiment, the most necessary helps to the finding out Causes and proportioning them to their Effects.

THEIR manner of Philosophizing was to give bodies certain *arbitrary* properties, such as

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best

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best serv'd their purpose in accounting for the Phænomena * of Nature; from whence proceeded so many various Sects of Philosophers; every one ascribing a different cause to the same appearance, as his particular genius and imagination led him.

THE chief agreement observable among most of the Ancient Philosophers consists in this, *viz.* that they conceived all bodies as compositions of Air, Earth, Fire, and Water, or some one or more of these, from whence they acquired the name of Principles or Elements, which they still retain.

EPICURUS advanc'd a little farther, and asserted, that though. bodies consisted of some one or more of these, yet that they were not strictly Elements, but that they themselves consisted of Atoms; by an accidental concourse of which, (as they were moving through infinite space in lines nearly parallel) all things received their Form and manner of Existence †.

DES CARTES has contrived an Hypothesis very different from the rest, he sets out with a sup-

* By a Phænomenon of Nature is meant any motion or situation of bodies among one another, which offers it self to the notice of our senses, and is not the immediate result of the action of an intelligent Being.

† For the Opinions of the Ancient Philosophers consult *Diogenes Laertius* and *Stanley's Lives*.

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position that the Universe at first was entirely full of matter, that, from this matter when first put in motion, there would necessarily be rubbed off (by the grinding of the several parts one against another) some particles sufficiently fine to pass through the hardest and most solid bodies without meeting with any resistance: of these consists his *Materia subtilis*, or *Materia primi Elementi*. He imagined that from hence also would result other particles of a globular form, to which he gave the name of *Materia secundi Elementi*. Those which did not so far lose their first figure, as to come under the denomination of *Materia primi* or *secundi Elementi*, he call'd *Materia tertii Elementi*; and maintain'd that all the variety, which appears in natural bodies was owing to different combinations of those Elements.

HE likewise supposes that God created a certain quantity of Motion and allotted it to this mass of Matter, which therefore (being created) can no more be annihilated without an omnipotent hand, than Body it self; in consequence of which he was obliged to teach, that the quantity of motion is always the same: so that if all the Men and Animals in the World were moving, yet still there would be no more motion, than when they were at rest; the motion lost

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being transfer'd to the *Æther*. So unaccountable are the notions of this great Philosopher, that it is surprizing his doctrine should have met with such universal reception, and have got so strong a party of Philosophers on its side, that notwithstanding it was more absurd, than the Schoolmens *Substantial Forms*, they must all be exploded to make way for his ingenious Hypothesis.

DES CARTES has been said by a late Writer* to have joyned to his great genius an exquisite skill in Mathematics, and by mixing Geometry and Physics together to have given the World hopes of great improvements in the latter. But this Writer ought to have been told that what he look'd upon in *Des Cartes's* Book of Principles as Demonstrations, are only Illustrations, there not being a Demonstration from Geometry in all his Philosophical Works†.

THE present method of Philosophizing establish'd by Sir *Isaac Newton* is to find out the Laws of Nature by experiments and observations. To this, with a proper application of Geometry, is owing the great advantage the

* Mr. *Wotton* in his *Reflections on Ancient and Modern Learning*.

† See this Subject discuss'd more at large in *Keil's* Introduction to his Examination of Dr. *Burnet's* Theory &c. Second Edition.

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present System of Philosophy has over all the preceding ones, and the vast improvement it has received within the last Age. It is indeed in vain to imagine, that a System of Natural Philosophy can be framed by any other method: for without observations it is impossible we should discover the Phænomena of Nature, without experiments we must be ignorant of the mutual actions of bodies, and without Geometry we can never be certain whether the causes assign'd be adequate to the effects we would explain, as the various Systems of Philosophy built on other foundations evidently shew.

THIS way of searching into Nature was first proposed by my Lord *Bacon**, prosecuted by the *Royal Society*, the *Royal Academy* at *Paris*, the honourable Mr. *Boyle*, Sir *Isaac Newton*, &c.

WHAT wonderful advancement in the knowledge of Nature may be made by this method of enquiry, when conducted by a genius equal to the work, will be best understood by considering the discoveries of that excellent Philosopher last mentioned. To him it is principally owing, that we have now a rational System of Natural Philosophy; 'tis he who, by pursuing the sure and unerring method of rea-

* See his *Novum Organum*.

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soning from experiment and observation, joyn-
ed with the most profound skill in Geometry,
has carried his enquiries to the most minute
and invisible parts of matter, as well as to the
most remote bodies in the Universe, and has
establish'd a System, not subject to the uncer-
tainty of a mere Hypothesis, but which stands
upon the secure basis of Geometry it self.



C H A P.



The Properties of Body.

IT being the design of *Physics* or Natural Philosophy to account for the Phænomena of the Material World, it must be our method to begin with laying down the known properties of Body.

THESE are 1. Solidity. 2. Extension. 3. Divisibility. 4. A capacity of being moved from place to place. 5. A Passiveness or Inactivity. These are all the essential properties of Bodies that we are acquainted with, and that they are essential, appears from what follows.

1. SOLIDITY, called also Impenetrability, is that power which Body has of excluding all others out of its place.

THAT Body, as such, must be endued with this property follows from its nature, for otherwise two bodies might exist in the same place, which is absurd. The softest are equally solid with the hardest, for we find by experiment, that the sides of a Bladder filled with Air or Water, can by no means be made to come together*.

* At Florence a hollow Globe of Gold was fill'd with Water, and then exactly clos'd; the Globe thus clos'd was put into a Press driven by the force of Screws; the Water finding no room for a nearer approach of its particles toward each other, made its way through the pores of that close Metal standing in drops like Dew on the outside, before the Globe would yield to the violent Pressure of the Engine. V. *Locke's Essay* B. 2. c. 4.

2. THAT

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2. THAT Body is extended, is self evident, it being impossible to conceive any Body which has not length, breadth and thickness, that is, Extension.

3. It is no less evident that Body is divisible, for since no two particles of Matter can exist in the same place, it follows that they are really distinct from each other, which is all that is meant by being divisible.

IN this sense the least conceivable particle must still be divisible, since it will consist of parts, which will be really distinct *. To illustrate this by a familiar Instance: Let the least imaginable piece of matter be conceived lying on a smooth plane surface, 'tis evident the surface will not touch it every where, those parts therefore, which it does not touch, may be supposed separable from the rest, and so on as far as we please; and this is what is meant when we say matter is infinitely divisible.

* This Proposition is demonstrated Geometrically thus, suppose the line *AD* (*Fig. 1.*) perpendicular to *BF* and another as *GH* at a small distance from it also perpendicular to the same line; with the Centers *CCC* &c. describe Circles cutting the line *GH* in the points *e, e, e*, &c. Now the greater the Radius *AC* is, the less is the part *eH*. But the Radius may be augmented in infinitum, and therefore the part *eH* may be diminished in the same manner; and yet it can never be reduc'd to nothing, because the Circle can never coincide with the right line *AF*; consequently the parts of any magnitude represented by *GH* may be diminished in infinitum. *Q. E. D.* V. *Keil's* Introd. ad *Phyf. Præl.* 3, 4, 5. *Gravesande's* Elem. Math. *Phyf. L.* 1. c. 4. Schol.

How

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How far matter may actually be divided, may in some manner be conceiv'd from hence *, that a piece of Wire, gilt with so small a quantity as 8 grains of Gold, may be drawn out to the length of 13000 feet, the whole surface of it still remaining cover'd with Gold †.

A quantity of Vitriol being dissolved and mix'd with 9000 times as much Water, will tinge the whole, consequently the Vitriol will be divided into as many parts as there are visible portions of matter in that quantity of Water ‡.

THERE are Perfumes, which, without a sensible diminution of their quantity, shall fill a very large space with their odoriferous particles, which must therefore be of an inconceivable smallness, since there will be a sufficient number in every part of that space, sensibly to affect the organ of smelling.

4. THAT all matter is moveable follows from its being finite: and to suppose it positively

* We have a surprizing instance of the minuteness of some parts of Matter from the nature of Light and Vision. Let a Candle be lighted and placed in an open plane, it will then be visible two miles round, consequently was it placed two miles above the surface of the Earth, it would fill with luminous particles a Sphere, whose diameter was four miles, and that before it had lost any sensible part of its weight. The force of this Argument will appear better when the Reader is acquainted with the cause of Vision.

† *Keil's* Introd. ad *Phyf. Præl.* 5. *Religious Philos. Contempl.* 25.

‡ *Mem. de l' Acad.* 1706.

infinite is absurd, because it consists of parts *.

5. BY the Passiveness or Inactivity of matter, (commonly call'd its *Vis Inertia*) is meant the propensity it has to continue its state of Motion or Rest, till some external force acts upon it. This will be farther explain'd under the first Law of Nature.

CH A P. II.

Of Vacuum.

I. **P**LACE void of Matter is call'd empty Space or *Vacuum*.

II. It has been the opinion of some Philosophers, particularly the *Cartesians*, that Nature admits not a *Vacuum*, but that the Universe is entirely full of Matter, in consequence of which opinion they were oblig'd to assert, that if every thing contain'd in a vessel could be taken out or annihilated, the sides of that vessel, however strong, would come together; but this is contrary to experience, for the air may be drawn out of a vessel by means of the Air Pump, which will nevertheless remain whole, if its sides are strong enough to support the weight of the incumbent Atmosphere.

III. SHOULD it be objected here, that it is impossible to extract all the Air out of a Vessel, and

* See Mr. Law's Translation of ABp. King *de Origine Malt*
Note 3.

that

that there will not be a Vacuum on that account; the answer is, that since a very great part of the Air that was in the Vessel may be drawn out, as appears by the quick descent of light Bodies in a Receiver*, there must be some vacuities between the parts of the remaining Air: which is sufficient to constitute a *Vacuum*. Indeed to this it may be objected by a *Cartesian*, that those vacuities are fill'd with *Materia subtilis* that passes freely through the sides of the Vessel, and gives no resistance to the falling Bodies; but since the existence of this same *Materia subtilis* can never be prov'd, we are not oblig'd to allow the objection, especially as Sir *Isaac Newton* has found, that all Matter affords a resistance nearly in proportion to its density †.

THERE are many other Arguments to prove this, particularly the motions of the Comets through the Heavenly Regions without any sensible resistance; the different weight of Bodies of the same bulk &c. but those being not yet explain'd are not so proper to be insisted on in this place.

* By this Term is meant any Vessel, out of which we extract the Air by the Air Pump.

† *Newtoni Opt.* p. 310.

C H A P. III.

Of Attraction and Repulsion.

BESIDES the forementioned properties of Matter, it has also certain powers or active Principles, known by the names of *Attraction* and *Repulsion*, probably not essential or necessary to its existence, but impressed upon it by the Author of its Being, for the better performance of the Offices for which it was design'd.

II. *ATTRACTION* is of two kinds. 1. *Cohesion*, or that by which the several particles whereof Bodies consist, mutually tend toward each other. 2. *Gravitation*, or that by which distant Bodies act upon each other.

III. *THE Attraction of Cohesion* is prov'd from abundance of Experiments, of which some of the most obvious are as follows.

1. LET a small glass Tube (commonly call'd a Capillary Tube) open at both ends, be dipt into a Vessel of Water, the Water will immediately rise up in the Tube to a certain height above the level. This rise of the Water is manifestly owing to the *Attraction* of those particles of the glass which lie in the inner surface of the Tube immediately above the Water : accordingly the quantity of Water raised

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raised is always proportionable to the largeness of that surface*.

2. LET two spheres of Quicksilver be placed near each other and they will immediately run together and form one globule.

IV. THE Laws of this Attraction are 1st. That it acts only upon contact or at very small distances, for the Spheres mentioned in the last experiment will not approach each other till they are plac'd very near. 2. It acts according to the breadth of the surfaces of the Attracting Bodies, and not according to their quantities of Matter. Let there be two polish'd glass Plates laid one upon another in such a manner, as to touch at one end, and there make a very small angle: if two unequal drops of oil be put between these plates at equal distances from the line of contact, so that the least may touch both glasses, they will then both move towards the ends that touch, because the Attraction of the

* The heights the Water rises to in different Tubes, are observ'd to be reciprocally as the diameters of the Tubes, from whence it follows that the quantities raised are as the surfaces which raise them.

Dem. Let there be two Tubes, the diameter of the first double to that of the second, the Water will rise half as high in the first as in the second, now was it to rise equally high in both, the quantity in the first would be four times as great as in the second, (Cylinders of equal heights being as the squares of their diameters; 11. *El.* 14.) therefore since it is found to rise but half as high, the quantity is but twice as much, and therefore as the diameter; but the surfaces of Cylinders are as their diameters, therefore the quantities of Water raised are also as the surfaces. Q. E. D.

See a Dissertation on this Subject. Part II.

surfaces

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surfaces inclines that way; but the largest, touching the glasses in most points, will move the fastest. 3. 'Tis observ'd to decrease much more than as the squares, of the distances of the Attracting Bodies from each other increase: that is, whatever the force of Attraction is at a given distance, at twice that distance it shall be more than 4 times less than before *.

V. FROM hence it is easy to account for the different degrees of hardness in Bodies; those whose constituent particles are flat or square, and so situated as to touch in many points, will be hard; those particles which are more round and touch in fewer points will constitute a softer Body; those which are spherical will form a fluid †.

VI. ATTRACTION of Gravitation is that by which distant Bodies act upon each other. This is seen every day in the falling of heavy Bodies toward the Earth.

VII. THE Laws of this Attraction are 1. That it decreases as the squares of the distances between the Centers of the attracting Bodies increase. Thus a Body at the surface of the Earth (*i. e.* about the distance of 4000 miles from its Center,) which weighs 10 Pounds, if it was plac'd 4000 miles above the surface of the Earth *i. e.* twice as far distant from the Center as before,

* V. Keilii Opera Ed. 4to. p. 626.

† Newtoni Optic. p. 335.

would

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would weigh 4 times less, if thrice as far, 9 times less &c. The truth of this Proposition is not to be had from Experiments, (the utmost distance we can convey Bodies to, from the surface of the Earth, bearing no proportion to their distance from its Center,) but is sufficiently clear from the Motions observ'd by the Heavenly Bodies. 2. Bodies attract one another with forces proportionable to the quantities of Matter they contain; for all Bodies are observ'd to fall equally fast in the exhausted Receiver, where they meet with no resistance. From whence it follows, that the action of the Earth upon Bodies is exactly in proportion to the quantities of Matter they contain; for was it to act as strongly upon a less Body as upon a larger, the least Body being most easily put into motion would move the fastest. Accordingly it is observable that the weight of a Body is the same, whether it be whole or ground to powder*.

VIII. FROM hence it follows, that was a Body to descend from the surface toward the Center of the Earth, it would continually become lighter and lighter, the parts above attracting it as well as those below, in which case it is demonstrated by the Mathematicians that the Gravity would decrease with the distance of the Body from the Center†.

* *Gravesande* Lib. 4. Chap. 11. *Cotes's* Preface to *Newton's* Princip.

† *Dem.* Let there be a Body as *P*, (*Fig. 2.*) placed any where within

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Scholium. IT may be proper to observe here, that when Philosophers speak of Bodies gravitating to, or attracting each other, that Body is said to gravitate to another, which moves towards it, while the other actually is, or appears to be at rest, and this other is said to attract the former; though indeed the force be-

within a concave sphere, as AB , which let us suppose divided into an infinite number of thin concentric surfaces; I say the Body P will be attracted equally each way by any one of these, *v.g.* the interior $IHKL$. Let there be lines as IL, HK , &c. drawn through any point of the Body P , in such a manner as to form the surface of two similar figures, suppose Cones, the diameters of whose bases may be IH, KL , which let be infinitely small. These bases (being as the squares of the lines IH, KL) (20. *Elem.* 6.) will be directly as the squares of their distances from P (for the Triangles being infinitely small are similar.) But those bases include all the particles of matter in the interior surface, that are opposite to each other; the opposite attractions are therefore in the same ratio with those bases, that is as the squares of the distances PH, PI . But the attraction is inversely as the squares of the distances of the attracting Bodies, §. 7. *i.e.* inversely as the squares of the same distances PK, PI ; these two ratios destroying each other, it is evident, that if the concavity of the Sphere was fill'd with Matter, that alone, which lies nearer the Center than the Body, can affect it, the respective actions of all the parts, that are more distant, being equal, and in contrary directions, since the same is demonstrable of any of the remaining concentric surfaces. Let us then see what effect that which lies nearer the Center than the Body will have upon it, which may be considered as a Sphere, on whose surface the Body is plac'd. The distances of each particle of Matter from the Body, (taken collectively) will be as the diameter of the Sphere, or as the Radius, *i.e.* as the distance of the Body from the Center; their action therefore upon the Body will be inversely as the square of that distance: & the quantity of Matter will be as the cube of that distance; (18. *Elem.* 12.) the attraction therefore will be also in that proportion. Now, these two ratios being compounded, the attraction will be only as the distance of the Body from the Center. *Q. E. D.*

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ing mutual and equal on both sides (as will be explain'd under the 3^d Law of Nature) the same Term might be apply'd to either the gravitating or attracting Body.

It is farther to be observ'd, that when we use the Terms, Attraction or Gravitation, we do not thereby determine the Physical Cause of it, as if it proceeded from some supposed occult quality in Bodies; but only use those Terms to signify an Effect, the Cause of which lies out of the reach of our Philosophy. Thus we may say, that the Earth attracts heavy Bodies; or that such Bodies tend or gravitate to the Earth: though at the same time we are wholly ignorant whether this is effected by some power actually existing in the Earth or in Bodies, or external to both: since it is impossible any error in our reasonings can follow from hence; it being evident, that all the consequences of such tendency must be the same, let the cause be where or what it will.

X. REPULSION is that property in Bodies, whereby if they are placed just beyond the Sphere of each others Attraction or Cohesion, they mutually fly from each other.

Thus if an oily Substance lighter than Water be placed on the surface thereof, or if a piece of Iron be laid on Mercury, the surface of the fluid will be depress'd about the Body laid on it. This depression is manifest-

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ly

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ly occasion'd by a repelling power in the Bodies, which hinders the approach of the Fluid towards them.

BUT it is possible in some cases to press or force the repelling Bodies into the Sphere of one anothers attraction; and then they will mutually tend towards each other, as when we mix Oyl and Water till they incorporate*.

XI. BESIDES the general Powers forementioned there are some Bodies that are endued with another call'd *Electricity*. Thus Amber, Jet, Sealing-Wax, Agate, Glass and most kinds of Precious Stones attract and repel light Bodies at considerable distances.

THE chief things observable in these Bodies are. 1. That they don't act but when heated. 2. That they act more forcibly when heated by rubbing than by fire. 3. That when they are well heated by rubbing, light Bodies will be alternately attracted and repell'd by them, but without any observable regularity whatever. 4. If a line of several yards in length has a Ball or other Body suspended at one end, and the other end be fixed to a glass Tube; when the Tube is heated by rubbing, the Electrical Virtue of the glass will be communicated from the Tube to the Ball, which will attract and repell light Bodies in the same manner as the

* We have an undeniable Proof of this Repulsive Force in Sir Isaac Newton's Opticks. B. 3. and Query 31.

glass it self does. 5. If the glass Tube be emptied of Air, it loses its Electricity*.

XII. LASTLY, the Loadstone is observ'd to have Properties peculiar to it self, as that by which it attracts and repels Iron, the Power it communicates to the Needle and several others†.

C H A P. IV.

Of the Laws of Motion commonly called Sir Isaac Newton's Laws of Nature.

I. **A**LL Bodies continue their state of rest or uniform motion in a right line, till they are made to change that state by some external force impressed upon them.

THIS Law is no other than that universal property of Bodies, call'd Passiveness or Inactivity; whereby they endeavour to continue the State they are in, whatever it be. Thus a Top only ceases to run round on account of the resistance it meets with from the Air, and the friction of the plane whereon it

* See *Hauksbee's Experiments. Philosoph. Transact.* Numb. 326.

† Several solutions of these Properties of *Electricity* and *Magnetism* have been attempted by different Philosophers, but all of them so unsatisfactory as not to deserve a particular account in this Place. See *Chambers's Dictionary in Electricity*, and *Des Cartes Opera Philosophica.* P. IV. §. 133. with several others cited in *Quaestiones Philosophica.*

moves. And a Pendulum when left to vibrate in *vacuo*, where there is nothing to stop it but the friction arising from the motion of the pin on which it is suspended, continues to move much longer than one in the open Air.

II. THE change of Motion, produc'd in any Body, is always proportionable to the force, whereby it is effected; and in the same direction, wherein that force acts.

THIS is an immediate consequence of this Axiom, the Effect is always proportionable to its Cause. For instance, if a certain force produces a certain motion, a double force will produce double the motion; a triple force triple the motion &c. If a Body is in motion, and has a new force impressed on it in the direction wherein it moves, it will receive an addition to its motion proportional to the force impressed; but if the force acts directly contrary to its motion, the Body will then lose a proportional part of its motion: again, if the force is impressed obliquely, it will produce a new direction in the motion of the Body, more or less different from the former in proportion to its quantity and direction. *

* This case is expressed more accurately by Mathematicians thus. If the proportion and direction of 2 forces acting upon a Body at the same time, be represented by the sides of a Parallelogram, the Diagonal of that Parallelogram will represent the proportion and direction of their united forces.

Dem. Let the Body *A* (Fig. 3.) be impell'd with a force, which would carry it to *E*, in the same time that another, acting upon

III. REACTION is always contrary, and equal to Action; or the actions of two Bodies upon each other are equal, and in contrary directions.

THUS, suppose a Stone or other Load to be drawn by an Horse; the Load reacts upon the Horse as much as the Horse acts upon the Load; for the harness, which is stretch'd equally between them both ways, draws the Horse towards the Stone, as much as the Stone towards the Horse, and the progressive motion of the Horse is as much retarded by the Load, as the motion of the Load is promoted by the endeavour of the Horse*. This will

upon it in the direction AD , would carry it to D . Imagine, that while the Body passes to E , the line AD (in which the Body moves by the other force) moves to EB , in a direction parallel to it self; when the Body has advanc'd to G in the line AE , the line AD will have got to GF , and the Body will have passed over such a part of it GH , as bears the same proportion to the whole line GF , as AG does to AE , that is GH (the shorter side of the Parallelogram GM , is to GF , or, which is the same thing, to EB (the shorter side of the Parallelogram ED), as AG (the longer side of the former) is to AE (the longer side of the latter,) from whence the Parallelograms are similar, *El. 6. Def. 1.* and consequently, by 24. *El. 6.* the point H is in the Diagonal, that is, the Body will always be found in the line AB . Q. E. D.

Coroll. From hence we have an easy method of resolving a given motion into any two, or more directions whatever; viz. by describing a Parallelogram about the given direction as a Diagonal, the two sides of which will represent the directions sought. Thus, suppose a Body was impell'd in the Line AB , we may conceive it as acted upon by two forces at the same time, one towards E , the other towards D , or any other two whatever, provided the lines be drawn of such length, that, when the Parallelogram is compleated, the given line AB shall be its Diagonal.

* It may be thought perhaps that (two equal and contrary forces

be better explain'd from the following instance; let a Person sitting in a Boat draw another equally heavy towards him, they will both move towards each other with equal velocities; let the Boat he sits in be the lightest, and it will move the fastest; because the action being equal on both sides, the same quantity of motion will be given to each Boat, that is, the lesser will have the greater velocity*.

WE have a farther confirmation of this from Attraction. Suppose two Bodies attracting one another, but hinder'd from meeting by some other Body placed between them: if their tendencies towards each other are not equal, then the Body that is between them, will be pressed on one side more than on the other, and consequently the stronger pressure overcoming the weaker, they cannot remain at rest, but will all move on continually in that direction wherein the stronger force acts; which is both contrary to the first Law of Nature and Experience. This may be try'd with a Loadstone and Iron; which, being put into proper Vessels contiguous to one another and made to

forces destroying one another) the Horse will in this case not be able to move at all, because the Load draws him back, as much as he draws the Load forwards. But it is to be observ'd that the strength of the Horse is not properly exerted upon the Load but upon the Ground; consequently the Ground reacting and continuing at rest pushes the Horse forward with just so much force as the Horse exerts, above what is counteracted by the Load.

* See the distinction between Motion and Velocity. Chap. 9.
float

float on water, will be an exact counterbalance to each other, and remain at rest, whatever be the attractive power of the Loadstone, or the proportion of their respective magnitudes.

THESE Laws receive an abundant additional proof from hence, *viz.* that all the conclusions that are drawn from them, in relation to the Phænomena of Bodies, how complicated soever their Motions be, are always found to agree perfectly with observation. The truth of which sufficiently appears in all parts of the *Newtonian* Philosophy*.

C H A P. V.

The Phænomena of Falling Bodies.

I. **T**HE Laws of Nature being thus explained, we proceed to account for those Phænomena, which are solvable by them.

II. To begin with those of Falling Bodies. Constant experience shews, that Bodies have a tendency towards the Earth, which is call'd Gravity, the Laws of which were enumerated in Chap. 3. §. 7.

III. THE height, Bodies can be let fall from, bears so small a proportion to their distance from the Center of the Earth, that it cannot

* See these Laws explain'd more at large by *Cheyne* in his *Principles of Philosophy*. *Keil's* Introd. ad *Phyf. Præl.* 11, 12.
fen-

sensibly alter their Gravity; which therefore may be conceiv'd, as acting constantly and uniformly upon them, during the whole time of their fall: from whence they must necessarily acquire at every instant, an equal degree of velocity, which on that account will constantly increase, in proportion to the time the Body takes up in falling.

IV. THE spaces Bodies fall through in different times, reckoning from the beginning of their fall, are as the squares of those times; thus, a Body will fall 4 times as far in 2 minutes, as it does in one, and 9 times as far in 3, 16 times as far in 4 &c. *

* In order to demonstrate this Proposition, it will be necessary to lay down the following Theorem. *viz.*

That the space a Body passes over, with an uniform motion, is in a ratio compounded of the time and velocity. For the longer a Body continues to move uniformly, the more space it moves over; and the faster it moves during any interval of time, the farther it goes; therefore the space is in a ratio compounded of both; that is, is had by multiplying one into the other.

Coroll. Therefore the area of a rectangle, one of whose sides represents the celerity a Body moves with, and the other the time of its motion, will express the space it moves through.

This being premised, let the line AB (Fig. 4) represent the time a Body takes up in falling, and let BC express the celerity acquir'd by its fall; farther, let the line AB be divided into an indefinite number of small portions, ei , im , mp , &c. and let ef , ik , mn , pq , &c. be drawn parallel to the base. Now it is evident from §. 3. (*viz.* that the velocities are as the times in which they are acquir'd) that the lines ef , ik , mn , pq , &c. being to each other (4. *El.* 6.) as the lines Ae , Ai , Am , Ap , &c. will represent the celerities in the times represented by these: that is, ef will be as the velocity of the Body in the small portion of time ei , and ik will be as the velocity in the portion of time im ; in like manner

V. FROM this Proposition it follows, that a Body falls 3 times as far, in the second portion of time, as it does in the first; 5 times as far in the third; 7 times in the fourth, and so on in the series of the odd Numbers: for otherwise, it could not fall 4 spaces in 2 minutes, and 9 in 3, as the Proposition asserts.

VI. THE spaces describ'd by Falling Bodies in different times are as the squares of the last acquir'd velocities. For by §. 4. the spaces are as the squares of the times, and by §. 3. the velocities are as the times; therefore the spaces are also as the squares of the velocities.

VII. THE space a Body passes over from the beginning of its fall in any determinate time, is half what it would describe in the same

pq will be as the velocity in the portion of time po , which portions of time being taken infinitely small, the velocity of the Body may be suppos'd the same, during any whole portion; and consequently, by the Corollary of the foregoing Theorem, the space run over in the time ei with the velocity ef may be represented by the rectangle if : in like manner the space run over in the time im with the celerity ik , may be express'd by the rectangle mk ; and that run over with the celerity mn in the time mp , by the rectangle pn ; and so of the rest. Therefore the space run over in all those times will be represented by the sum of all the rectangles, that is, by the triangle ABC , for those little triangular deficiencies, at the end of each rectangle, would have vanished, had the lines ei , im , mp , &c. been infinitely short, as the times they were supposed to represent. Now as the space, the Body describes in the time AB , is represented by the triangle ABC , for the same reason the space pass'd over in the time Ao may be represented by the triangle Aor , but these triangles being similar are to each other, as the squares of their homologous sides AB and Ao (20. *El.* 6.): that is, the spaces represented by the triangles are to each other, as the squares of the times represented by the sides. *Q. E. D.*

D

time

time moving uniformly with its last acquir'd velocity*.

VIII. IN like manner, when Bodies are thrown up perpendicularly, their velocities decrease, as the times they ascend increase; their gravity destroying an equal portion of their velocity every instant of their ascent.

IX. THE heights Bodies rise to, when thrown perpendicularly upwards, are as the squares of the times spent from their first setting out, to the moment they cease to rise. That is, if a Body is thrown with such a degree of velocity, as to continue rising twice as long as another, it shall ascend 4 times as high; if thrice, 9 times as high, &c.

THESE two are the converse of the 3^d and 4th Sections†.

C H A P. VI.

Of the descent of Bodies on oblique Planes, and of Pendulums.

WHEN a Body descends on an oblique Plane, its motion is continually acce-

* For let the time be AB , and the last velocity BC , the space the Body runs over, while it is acquiring that velocity, is ABC , but the space it would pass over in the time AB , was it to move uniformly with the celerity BC , is, by the Theorem, (Note p. 24.) the space $ABCD$, double the former. Q. E. D.

† See Keil's Introd. ad Phys. Præl. 11. *Gravesande* L. 1. Ch. 17.

lerated

lerated by the action of gravity, but in a less degree, than when it descends perpendicularly; its free descent in this case being hinder'd by the interposition of the Plane: from whence it follows, that what was said in the last Chapter, concerning the perpendicular descent of Bodies, is true of such as fall on oblique Planes, allowance being made for the difference of acceleration.

II. THE effect of Gravity upon a Body falling down an oblique Plane, is as much less than the same acting on another falling freely; as the perpendicular height of the plane is less than its length*.

III. A Body falls through as much longer space perpendicularly, than it does obliquely in the same time, as the oblique side of the Plane is longer than the perpendicular height†.

* *Dem.* Let AC (Fig. 5.) be the inclin'd Plane, the Body at A , and the action of gravity, whereby it endeavours to fall perpendicularly, represented by the line AB ; let AD be perpendicular to AC , AD will then represent the direction by which the Plane acts upon the Body (for all Bodies act in lines perpendicular to their surfaces,) let then those two forces be resolved into one in the direction AC , (as shewn in Note to §. 4. Chap. 4) by completing the Parallelogram BD whose Diagonal will be AG . In order to this BG must be let fall perpendicularly upon AC (that it may be parallel to the opposite side of the Parallelogram AD) consequently (8. *Elem.* 6) AG is to AB as AB to AC , that is, the tendency of the Body down the plane is to its perpendicular tendency, as AB is to AC . *Q. E. D.*

† That is, supposing BG (Fig. 5.) perpendicular to AC , the Body would fall to G in the same time it would fall to B , for, as was observ'd Note the last, AB is as much longer than AG as AC is longer than AB .

IV. THE velocity a Body acquires by falling perpendicularly, as much exceeds that which it acquires by falling obliquely in the same time, as the space of its perpendicular descent in that time exceeds that of its oblique one*.

V. A Body takes up as much more time in falling down the oblique side of a Plane, than it does the perpendicular height of it, as the oblique side exceeds the height†.

Dem. It is evident that in all motions equally accelerated, the sum of the velocities produced in any time (that is the spaces run over in that time) are as the generating forces, that is, the space pass'd over by the Body on the oblique side AC , is to the perpendicular space a Body would pass over in the same time; as the action of gravity on the Body in the direction AC , to its perpendicular action in the direction AB : but these actions are to each other, (by §. the last,) as the oblique side to the perpendicular height, the spaces therefore pass'd over will be in the same proportion. *Q. E. D.*

* Since by §. the last, a Body falls to G , (*Fig. 5.*) in the same time another falls to B , and by (*Chap. 5. §. 7.*) the space a falling Body passes over in any time, is half that which it would run over in the same time moving uniformly with its last acquir'd velocity, it follows that the Body falling down the oblique plane would pass over double the space AG , moving uniformly with its last acquir'd velocity, in a portion of time equal to that in which it was acquir'd; likewise double the space AB would be pass'd over by the other Body, moving uniformly with its last acquir'd velocity, in a portion of time equal to that in which it was acquir'd; but since the velocities of Bodies moving uniformly are as the spaces they run over in equal times, the velocities of the Bodies in G and B are to each other as double the lines AG and AB , that is as the lines themselves, which by §. 3. are as the spaces run through in the same time, from whence the Proposition is clear.

† *Dem.* The square of the time in which AC (*Fig. 5.*) is run over, is to the square of the time in which AG is run over, as AC to AG , (by *Chap. 5. §. 4.*) that is, since AC , AB , AG are continually proportional (*8. Elem. 6.*) as the square of AC to the square

VI. A Body acquires the same velocity in falling down the oblique side of a Plane, as if it fell freely through the perpendicular height of it *.

VII. A Body takes up the same time in falling through the Chord of a Circle, whether it be long or short, as it does in falling perpendicularly through the diameter of the same Circle †.

VIII. UPON this is founded the Theory of Pendulums: for from hence it follows, that supposing a Pendulum could be made to vibrate in a Chord of a Circle, instead of an arch, all its vibrations would require the same time, whether they were large or small ‡.

square of AB (by *Def. 10. Elem. 5.*) therefore the times themselves are as the lines AC and AB , that is, as the oblique side of the Plane to the perpendicular height. *Q. E. D.*

* *Dem.* The square of the velocity a Body acquires by falling to G , is to the square of the velocity it acquires by falling to C , as the space AG to the space AC (by Chap 5. § 4.) that is (by 8. *Elem. 6.* and *Def 10. Elem. 5.*) as AGq to ABq . But since AG is run over in the same time AB is, (see Note to §. 3.) the velocity in G is to the velocity in B , as AG to AB , (by §. 4.) and consequently since the velocities both in C and B bear the same proportion to that in G , they must be equal to each other. *Q. E. D.*

† *Dem.* It was demonstrated (§. 3.) that a Body will fall from A to G , (*Fig. 6.*) on the inclin'd Plane AC , in the same time another would fall freely to B , provided AGB is a right angle, in which case AG (by 31. *Elem. 3.*) is a Chord of that Circle of which AB is the Diameter; therefore a Body falls through the Chord GC . *Q. E. D.*

‡ This may be illustrated by conceiving the last figure inverted (as in *Fig. 7.*), where supposing the Ball suspended in such a manner, as to swing in the right line GA instead of the Arch GA , it would always fall through it in the same time however long or short

IX. FROM hence we see the reason, why the shorter arches a Pendulum describes, the nearer its vibrations come to an equality, for small arches differ less from their Chords than large ones. But if the Pendulum is made to vibrate in a Curve, which Mathematicians call a *Cycloid*; each swing will then be perform'd in the same time, whether the Pendulum moves through a larger or lesser space. For the nature of this Curve is such, that the tendency of a Pendulum towards the lowest point of it, is always in proportion to its distance from thence; and consequently let that distance be more or less, it will always be run over by the Pendulum in the same time *.

short it was, for the inclination of the line GA to the horizontal line BC , is not alter'd by inverting the figure.

* The Description of a *Cycloid*.

Upon the right line AB , (*Fig. 8.*) let the Circle HDE be so plac'd, as to touch the line in the point H , then let this Circle roll along upon it from H to C , as a wheel upon the ground, then will the point H in one revolution of the Circle describe the Curve HKC , which is call'd a *Cycloid*. Now suppose two Plates of Metal bent into the form HK and KC , and placed in the situation LH and LC , in such manner, that the points H and C may be apply'd to L , and the points answering to K be apply'd to H and C . This done, if a Pendulum as LP , in length equal to LH , be made to vibrate between the Plates or Cheeks of the *Cycloid* LC and LH , it will swing in the line CKH ; and the time of each vibration, whether the Pendulum swings through a small or a great part of the *Cycloid*, will be to the time a Body takes up in falling perpendicularly through a space equal to IK , (half the length of the Pendulum;) as the Circumference of a Circle to its Diameter, and consequently it will always be the same.

They that would see a Demonstration of this and several other things relating to this Curve, may consult *Huygens Horol. Oscillatorium* or *Cotes's Harmonia Mensurarum*.

Fig. 4. P. 24.

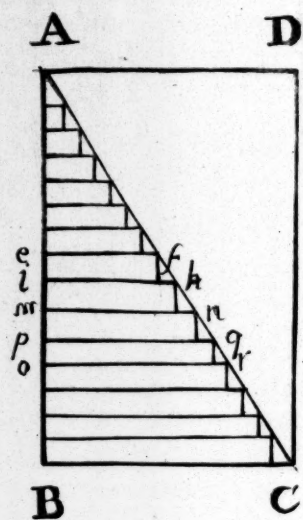


Fig. 5. P. 27.

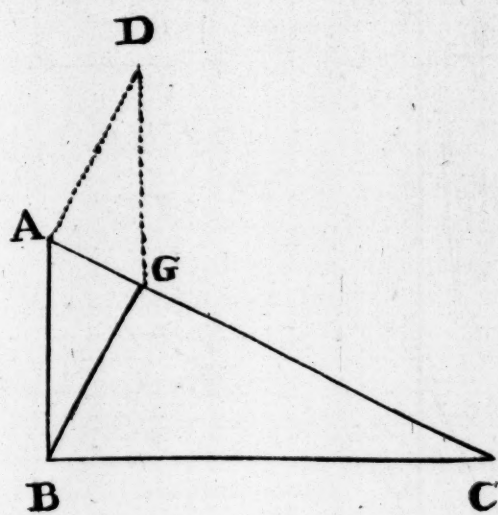


Fig. 6. P. 29.

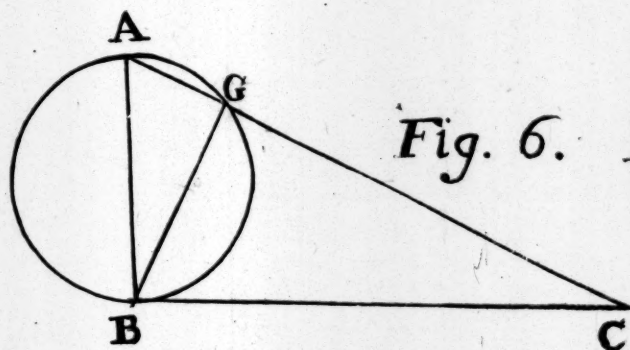
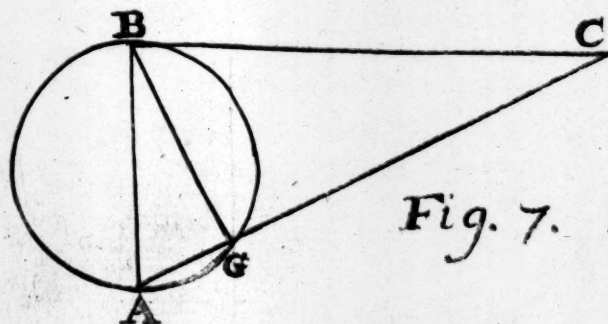


Fig. 7. P. 29.



X. THE time of the descent and ascent of a Pendulum, supposing it to vibrate in the Chord of a Circle, is equal to the time in which a Body falling freely would descend through eight times the length of the Pendulum.

FOR the time of the descent alone upon the Chord is equal to that in which a Body would fall through the Diameter of the Circle (by §. 7.); that is, twice the length of the Pendulum: but in twice that time (*viz.* during a whole vibration) the Body would fall four times as far (Chap. 5. §. 4.), that is, through eight times the length of the Pendulum.

XI. THE times, that Pendulums of different lengths perform their vibrations in, are as the square roots of their lengths*.

XII. THE Center of *Oscillation* is a point in which, if the whole gravity of a Pendulum was collected, the time of its vibration, would not be alter'd thereby†; this is the point from

* *Dem.* Let there be two Pendulums *A* and *B* (*Fig.* 9. and 10.) of different lengths, the time the first vibrates in (suppose through a Chord) is equal, to the time in which a Body would fall freely through *DA*, the Diameter of the Circle (as demonstrated §. 7.); in like manner the time *B* vibrates in, is that in which a Body would fall through *FB*. Now the times in which Bodies fall through different spaces are as the square roots of those spaces, that is, of *DA* and *FB*, or of their halves *CA* and *CB*, *i. e.* of the lengths of the Pendulums. *Q. E. D.*

† The Rule for finding the Center of Oscillation.

If the Globe *AB* (*Fig.* 11.) be hung by the string *CD*, whose weight is inconsiderable, the Center of Oscillation is found thus:
sup-

whence the length of a Pendulum is measur'd, which in our Latitude, in a Pendulum that swings seconds, is 39. 2. inches.

XIII. THE squares of the times in which Pendulums, acted upon by different degrees of gravity, perform their vibrations in, are to each other, as the gravities *. *inv.*

suppose *E* the Center of the Globe, take the line *G* of such a length, that it shall bear the same proportion to *ED* as *ED* to *EC*, then *EH* being made equal to $\frac{2}{5}$ of *G*, the point *H* shall be the Center of Oscillation

If the weight of the Rod *CD* be too considerable to be neglected divide *CD* (*Fig. 12.*) in *I*, so that *DI* may be equal to $\frac{1}{3}$ of *CD*, and make a line as *K*, in the same proportion to *CI*, that the weight of the Rod bears to that of the Globe, then having found *H* the Center of Oscillation of the Globe, as before, divide *IH* in *L*, so that *IL* may bear the same proportion to *LH*, as the line *CH* bears to the line *K*; then will *L* be the center of Oscillation of the whole Pendulum. See *Huygens* Horol. Oscillat. pag. 141, 142.

* *Dem.* The spaces falling Bodies descend through, are as the squares of the times, when the gravity by which they are impell'd is given (*Chap. 5. §. 4.*); and as the gravity when the time is given (for the sum of the velocities produced in any time will always be as the generating forces): consequently when neither is given, they are in a ratio compounded of both; the squares of the times are therefore inversly as the gravities.

[For if in 3 quantities *a*, *b*, *c*; *a* is as *bc*, then $b : \frac{a}{c}$, i. e. if *a* is given, as $\frac{1}{c}$ or as *c* inversly.] But if the squares of the times in which Bodies fall through given spaces are inversly as the gravities by which they are acted upon; then the squares of the times in which Pendulums of equal lengths, perform their vibrations, will be also in the same ratio, on account of the constant equality between the time of the vibration of a Pendulum, and of the descent of a Body through eight times its length (§. 12.)

FROM

Fig. 8. P. 30.

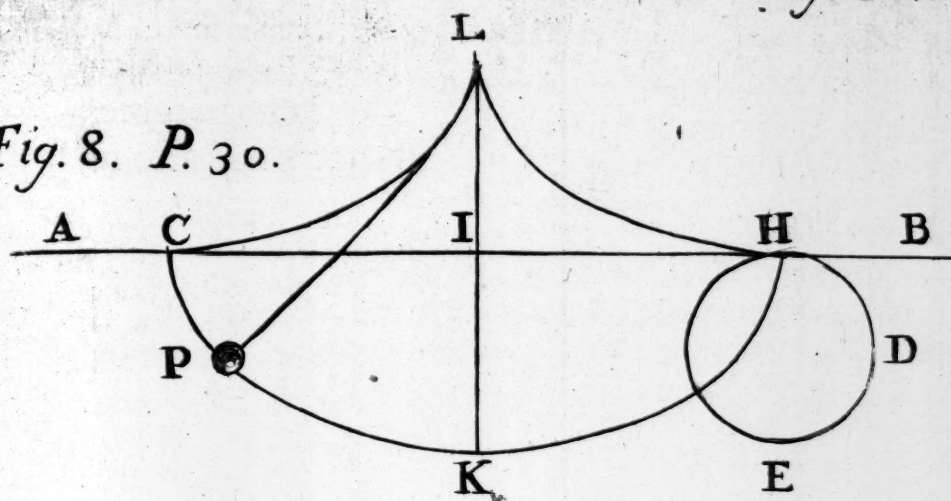


Fig. 9. P. 31.

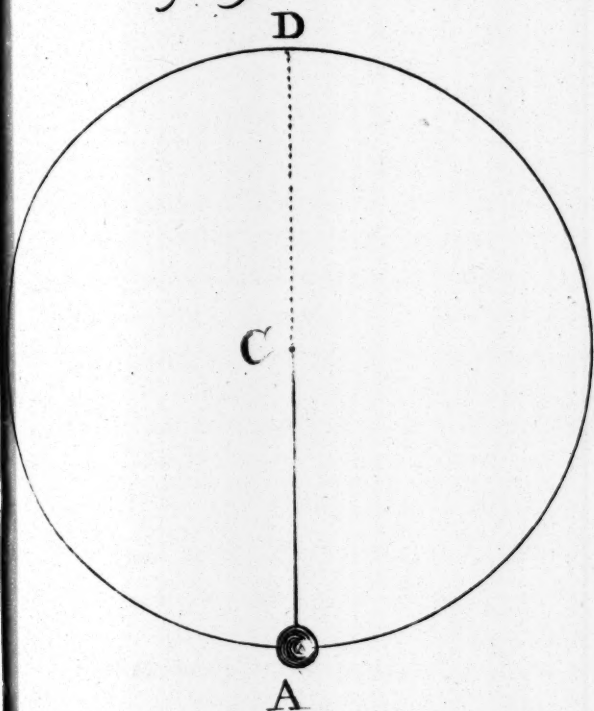
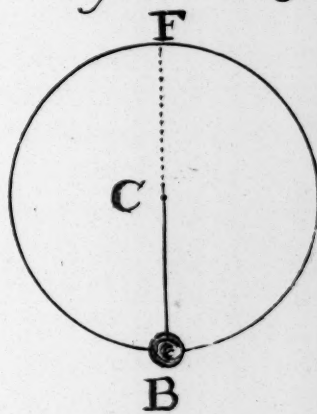


Fig. 10. P. 31.





FROM whence it follows, that a Pendulum will vibrate slower when nearer the Equator, than the same when nearer the Poles; for the gravity of all Bodies is less, the nearer they are to the Equator; *viz.* on account of the spheroidical Figure of the Earth, and its rotation about its Axis, as will be explain'd hereafter. To which we may add the increase of the length of the Pendulum occasion'd by the heat in those parts; (for we find by experiment that Bodies are enlarged in every dimension, in proportion to the degree of heat that is given them:) for which reason (Chap. 6. §. 11.) the vibrations of the Pendulum will also be slower.

C H A P. VII.

Of Projectiles.

I. **A** BODY projected in a direction parallel or oblique to the Horizon would proceed on in infinitum in a right line, (by the first Law of Nature) but being continually accelerated towards the Earth by its Gravity, it will describe a Curve called a *Parabola* *.

* *Dem.* Let us suppose the Body thrown from *A*, in the direction *AB* horizontally (*Fig. 13.*) or obliquely (*Fig. 14.*) it would (if not attracted towards the Earth) in equal times describe equal parts of the line *AB*, as *AC*, *CD*, *DE*, &c. but if in the first portion of time, while it moves from *A* to *C*, it descends by its Gravity as far as *G*, by a composition of these two Motions (Chap. 4. §. 2.)

II. THE greatest distance, to which a Body can be thrown with a given velocity, is at the elevation of 45 degrees*.

it will be found in H , and while it moves from A to D twice as far, it will move downwards to M , 4 times as far as before (Chap. 5. §. 4) and will therefore be found in I supposing DI 4 times as long as CH . Again, while it moves to E three times as far from A as C is, it will have moved downwards 9 times as far as it did in the first portion of time, and therefore will be found in K , provided EK be 9 times as large as CH &c. that is the lines CH , DI , EK , &c. will be to each other as the squares of the lines AC , AD , AE , &c. which is the property of the parabolic Curve, (De L' Hospital B. I. Prop. 1. Cor. 2. and Prop. 3. Cor. 1.) and consequently the line $AHIK$, &c. which the Body moves in, whether thrown horizontally or obliquely, is a Parabola. Q. E. D.

* It is demonstrated by the Writers on Conic Sections, that the Quotient which arises from the division of the square of the line GH by the line AG viz. the quantity $\frac{GH^2}{AG}$ (in either of the parabolic curves, (Fig. 13. or 14.), or of the square of MI by the line AM viz. $\frac{MI^2}{AM}$ or of the square of NK by AN viz. $\frac{NK^2}{AN}$ &c. provided those lines are all parallel to AB which touches the curve in the point A , is always the same: which Quotient is call'd the *Parameter* of the point A .

Now the velocity, with which the Body is projected from A , being (*ex hypoth.*) such as would carry it to C , in the time it would fall by its own gravity to G ; and to E in the time it would fall to N ; and since it would move over twice the space AN in that time, had it moved uniformly with the velocity acquir'd at N ; it follows, that the velocity it moves with from A to E , is to that which a Body acquires by falling to N , as AE to twice AN (Chap. 5. §. 7.) or as $\frac{1}{2} AE$ to AN . But the velocity a Body would acquire by falling through a fourth part

of the Parameter of the point A viz. $\frac{\frac{1}{4} NK^2}{AN}$ is to the velocity it would acquire by falling to N , as AE to $2AN$: (see this demonstrated in Note †) therefore the velocity a Body ought to be projected with from A to make it describe the given Parabola

$AHIK$

$AHIK$, is equal to the velocity it would acquire by falling through a fourth part of the Parameter belonging to that point of the Parabola from whence it is projected.

† The square of the velocity acquir'd by a Body in descending through a fourth part of the Parameter, or $\frac{\frac{1}{4}NKq}{AN}$ is to the square of that which is acquir'd by falling through the line AN ; as $\frac{\frac{1}{4}NKq}{AN}$ to AN , (Chap. 5. §. 6.), that is, multiplying both terms by AN , as $\frac{1}{4}NKq$ to ANq , and by extraction of their square roots, as $\frac{1}{2}NK$ to AN . Q. E. D.

Corol. This affords us an easy method of finding what direction it is necessary to throw a Ball in with a given velocity, in order to strike an object in a given situation. *v. g.* Let it be requir'd to strike an object as K with a ball thrown from A with a given velocity. Here it is only necessary to make the triangle ANK (suppose a right line drawn from A to K) such, that $\frac{NKq}{AN}$

or which is the same thing $\frac{AEq}{EK}$ in the triangle AEK , may be equal to four times the space a Body must fall through, to acquire such a degree of velocity as that with which it is intended to be thrown, and then AE will be the direction sought. In order to this we must lay down the following Lemma.

Let there be a Circle as ABC (Fig. 15.) AK a Tangent in the point A , AB perpendicular to the Horizon and parallel to KE or KI , I say $\frac{AEq}{EK} = AB$. For the angle ABE is equal to the angle EAK (32. Elem. 3.), and the angle BAE is equal to the angle AEK as alternate, therefore the triangles ABE and AEK are similar; consequently AB is to AE , as AE to EK , and multiplying the extreme terms together, and middle terms together, $AB \times EK = AEq$ and dividing both sides of the equation by EK , $AB = \frac{AEq}{EK}$. Q. E. D.

By the same method of arguing $\frac{AIq}{IK}$ may be proved equal to AB .

THE PROBLEM.

Let it be requir'd to strike an object as K (Fig. 16.) with a Ball projected from A with a given velocity.

E 2

Solution.

III. If 2 Balls are thrown at different elevations (but with equal degrees of velocity), the one as much above 45 degrees as the other below, the horizontal distances (or Ranges) where they both fall will be the same *.

Solution. Erect AB perpendicular to the Horizon, and equal to four times the height a Body must fall from, to acquire the velocity with which the Ball is to be thrown; bisect this in the point G , through which draw HC perpendicular to AB , and meeting the line AC (perpendicular to AK) in C . On C as a Center with the Radius CA , describe the Circle ABD ; lastly through K draw the line KEI perpendicular to the Horizon, cutting the Circle in the points E and I ; I say AE or AI will be the direction sought.

For by the Lemma $AB = \frac{AEq}{EK}$ or $\frac{AIq}{IK}$, but (*ex constructione*) AB is equal to four times the height a Body must fall from, to acquire the velocity with which it is to be thrown, therefore its equal $\frac{AEq}{EK}$ or $\frac{AIq}{IK}$ is the same, which by the Corollary was the thing requir'd to determine the direction sought, consequently the Parabola, which the Body will describe, will pass through the point K . Q. E. D.

Coroll. 1. From hence it is evident, that if the object to be struck, be placed any where in the horizontal line AO (*Fig. 17.*) beyond Q , the Problem is impossible; for then QH will not touch the Circle, and the Ball will not reach that point with any direction whatever. And that when the Ball is directed towards H , it will fall on Q the greatest distance it can possibly be thrown to; but the angle QAH being equal to ABH in the opposite segment (32. *Elem. 3.*) is equal to half AGH at the Center (20. *Elem. 3.*) which is a right one; consequently QAH is an angle of 45 degrees.

* *Coroll. 2.* If the object is situated in the horizontal line AO (*Fig. 18.*) but nearer to A , than the greatest horizontal distance at which it may be struck, suppose in K ; the two directions AE and AI with which it may be hit, are equally distant from the direction AH ; for the angles IAH and HAE are equal, as insisting on equal arches IH and HE (28. *Elem. 3.*)

IV. The height a Body will rise to, when thrown perpendicularly upwards, is equal to half the greatest horizontal distance it can be thrown to with the same velocity*.

FROM hence we may easily know how far a Mortar-Piece, or other such Machine, will carry a Ball. Let the Ball be thrown perpendicularly upwards, note the time of its ascent and descent, half that is the time of descent, from whence we learn the height, to which the Ball is thrown, for Bodies are observ'd to fall in the first second of time 16 feet, consequently in 2 seconds they fall 4 times 16 feet (Chap. 5. §. 4.) in 3, 9 times as much &c. but (§ 4.) the perpendicular height being doubled will give the greatest horizontal distance to which that Machine will carry the Ball with an equal Charge.

V. THE Randoms of two Projectiles, having the same degrees of elevation, but thrown with different velocities, are as the squares of the velocities: for by the last, the Randoms are as the heights to which the Bodies thrown perpendicularly upwards will ascend, but the

* Coroll. 3. The altitude of a perpendicular projection is equal to a fourth part of the height AB ; for the velocity with which the Body is projected, is (*ex hypoth.*) such as it would acquire by falling through a fourth part of the line AB ; but a fourth part of the line AB is equal to half the line GH , or AQ (Fig. 17.) that is half the greatest horizontal distance to which the Body can be thrown.

See Cotes's *Harmonia Mensurarum* p. 87. Keil's *Introduct. ad Phys. Præl.* 16.

heights are (Chap. 5. §. 6.) as the squares of the velocities.

VI. SUPPOSING the motion of the Earth, all Bodies, when thrown perpendicularly upwards, describe *Parabolas*; notwithstanding they appear both to ascend and descend in the same right line.

THIS may very easily be illustrated in the following manner; let there be a Body carried uniformly along the line AB (*Fig. 19.*) by the motion of the Earth from A towards B ; as it passes the point C let it be projected upwards by some force acting underneath it in the direction CO perpendicular to the former; the Body will not thereby lose its motion which it had in common with the Earth towards B (by the first Law of Nature), but will be carried by two motions, one towards B the other towards O ; let us then suppose, that in the time it would have advanced forwards to P in the line AB , it rises upwards to M in the line CO ; it will then be found in D (Chap. 4. §. 2.): in like manner supposing it would have advanced forward to Q while it rises to N , it would then be found in E , afterwards in F , then in G &c: describing the Curve CGI which (from what was demonstrated under §. 1.) is a Parabola*.

* *Dem.* Suppose the motion the Body had in common with the Earth towards B (*Fig. 20.*) and that with which it is projected towards O , such, as being compounded (Ch. 4. §. 2.) would have

Plate IV. Pag. 38.

Fig. 11. P. 31.

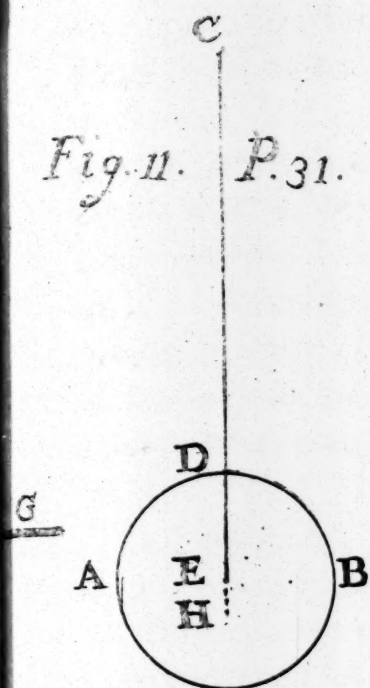


Fig. 12. P. 32.

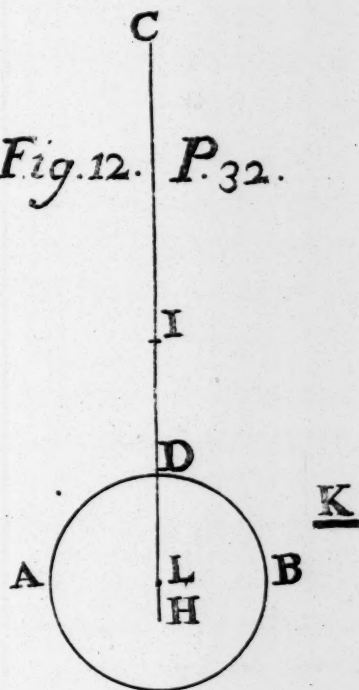


Fig. 13.

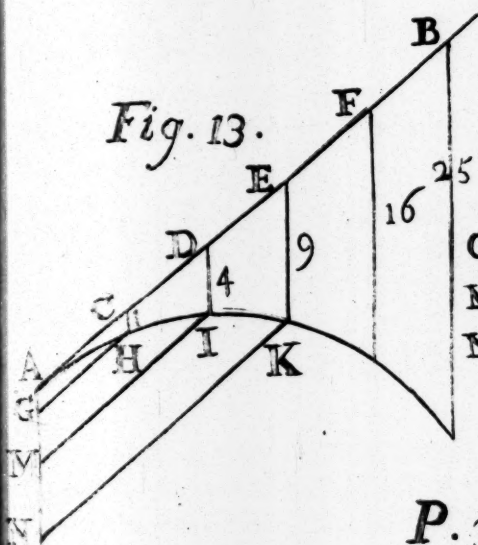
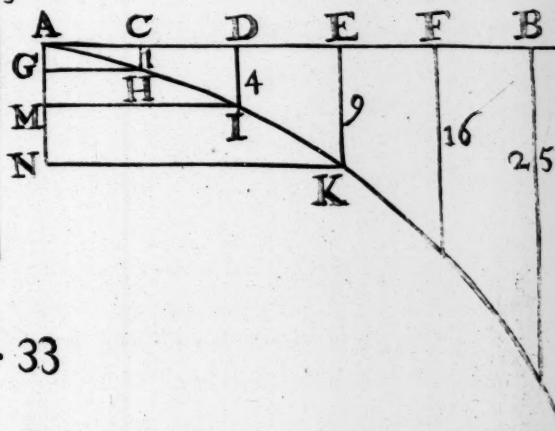


Fig. 14.





THE reason why it appears to a Spectator to rise and fall perpendicularly, is because he is carryed uniformly along with it by the motion of the Earth in its first direction. *v.g.* Suppose the Spectator at C at the instant the Body is thrown from thence, when it arrives at D, he will be moved to P, when the Body is at E he will be at Q &c. as is evident from what was observed about the motion of the Body in the Curve; and they will both meet in L. Therefore since the Spectator imagines himself standing still, and sees the Body always perpendicularly over his head, he must of course think that it rises right up and falls right down.

It may be proper to observe here, that Experiments relating to the motion of projected Bodies, do not exactly answer the Theory; the resistance of the Air destroying part of their motion: for which a small allowance is to be made.

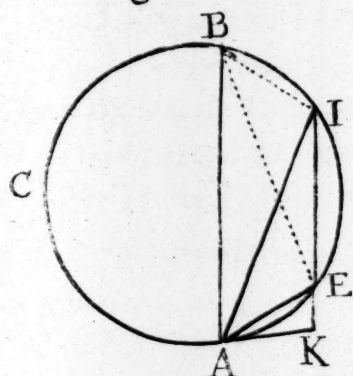
have produced a motion in the direction CX ; it will follow from thence, that the path described by it will be the same, as if it had been thrown in that direction from a point as C at rest; but in that case it would have describ'd a Parabola as CGL (§. 1.) therefore in this. *Q. E. D.*

C H A P. VIII.

Of Centripetal and Centrifugal Forces.

WHEN a Body is projected in an horizontal direction and by its Gravity made to describe a Parabola as demonstrated Chapter the last; the curvature of that Parabola will vary in proportion to the velocity with which the Body is thrown, and the Gravity which impels it towards the Earth. For the less its Gravity is in proportion to the quantity of matter it contains, or the greater the velocity is with which it is projected; the less will it deviate from a strait line, and the further it will go, before it falls to the Earth. For instance, if a Bullet be shot out of a Cannon from the top of a Mountain with a given velocity in an horizontal direction, and goes in a curve line, suppose to the distance of two Miles from the foot of the Mountain before it falls to the ground; the same Bullet shot with a much greater velocity would fly to a much greater distance before its fall. And by encreasing the velocity, the distance to which it is projected, may be encreased as much as you please; so that it will not fall to the ground, till it is arrived at the distance of ten, or thirty, or ninety degrees; or till it has even
sur-

Fig. 15.



P35.

Fig. 16.

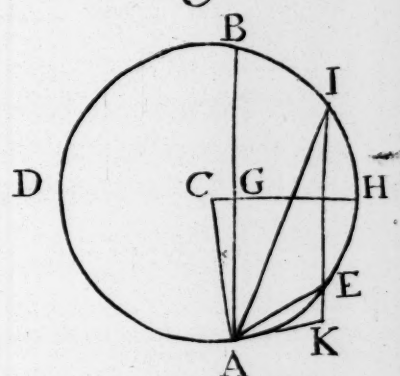
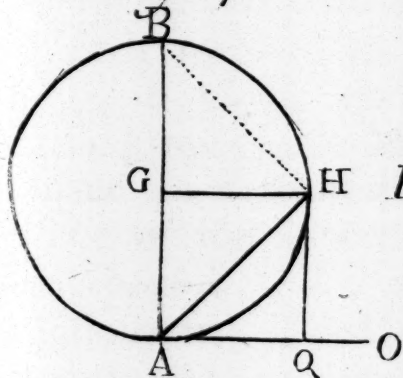


Fig 17



P36.

Fig 18

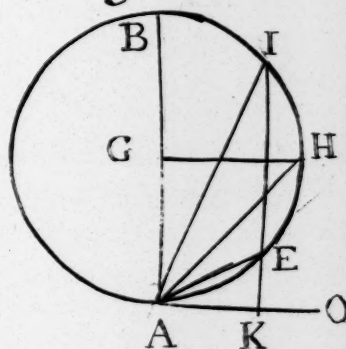
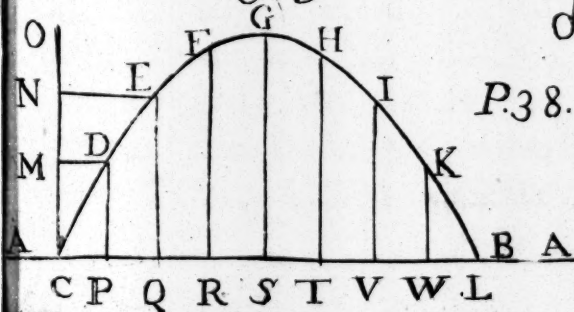
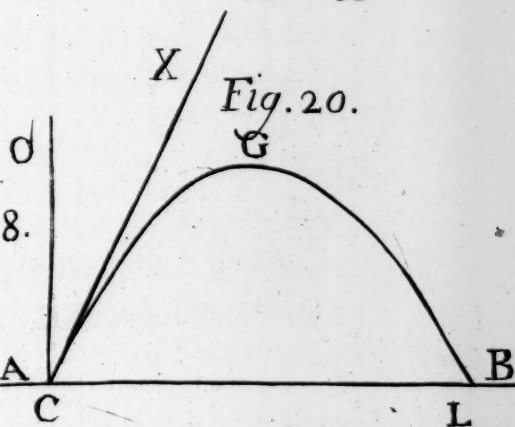


Fig. 19.



P38.

Fig. 20.





surrounded the whole Earth, and arrives at the very top of the Mountain from whence it was projected: in this case it will perform a second revolution, and so on *in infinitum* without a new projection, provided the resistance of the Air is taken away. Nay it may be projected with such violence, that it will continually recede from the Earth, moving in a Curve, till at length it gets out of the Sphere of the Earth's Attraction; after which it will go on in a straight line without ever returning. Which may thus be illustrated.

LET ABC (*Fig. 21.*) represent the Earth, M the point from whence the Body is projected in the direction MQ: it may be thrown with such force as to carry it to B before it falls, or to C, or even to go round to M, describing the Circle MDM; or lastly it may be made to describe the Curve MO, till it gets out of the Sphere of the Earth's Attraction, suppose at O, going on afterwards in the infinite strait line OX, there being nothing to stop or alter its course. Farther, it may be projected with such a force from M (*Fig. 22.*) as will cause it continually to recede from the Earth, till it arrives at the opposite point G, describing the curve MKG; and if the point G is within the Sphere of the Earth's Attraction, the Body will return to M, describing the Curve GLM exactly similar to MKG;

F
and

and in moving nearer and nearer to the Earth till it comes to M, will regain what velocity it lost in going from M to G, its Gravity conspiring with its motion from G to M in the same degree in which it opposed it from M to G; consequently the Body when at M having recovered the velocity with which it set out, will be inabled to perform a second revolution in the same Curve as before, and so on.

AGAIN, suppose it had been projected from the point M with a less degree of force than would have carryed it round in the Circle MDM (*Fig. 21.*), but greater than would have suffered it to have fallen to the Earth at the opposite point F (*Fig. 22.*); it would also in this case have arrived at the point M from whence it set out; for the excess of velocity it would have gained in F, by its tendency towards the Earth in its way thither, over and above that with which it was projected from M, would be sufficient to carry it off again from the Earth, till it arrived at M; and to make it describe the path FPM exactly similar and equal to the former, losing in its way from F to M just so much velocity, as it gained by passing from M to F; and thereby it would be inabled to perform an infinite number of revolutions in the same Curve without requiring a second projection.

FROM

FROM hence it follows, that supposing a Body projected from a point at any distance within the Sphere of the Earth's Attraction, with a force sufficient to carry it half round without falling to the surface, it is impossible it should fall upon any part of the other half; but will return to the point from whence it set out, making continual successive revolutions in the same Curve; provided it meets with no resistance from the Medium through which it passes, nor any other obstacle to obstruct its motion*.

FROM hence also it is clear, that, the nearer the revolving Body approaches to the Earth, the faster it moves; its velocity being continually increased during the time of its access towards the Earth, and as much retarded during its recess from it. And this acceleration and retardation will always be such, that the Body will describe equal Areas in equal times: the meaning of which is, that if we imagine a line constantly extended from the Center of the Earth to the Center of the Body, that line will always describe or pass through equal surfaces or spaces in equal times, for it constantly

* Gravity is here supposed to be inversely as the squares of the distances from the Earth, for 'tis possible that the force by which a Body tends towards another, may vary in such a manner at different distances, that the projected Body shall describe a Spiral line, continually approaching to or receding from that about which it revolves.

becomes shorter the faster it moves, and *vice versa* *.

AND for the same reason that a Body projected with a sufficient velocity may by the force of Gravity be made to describe a Curve round the Earth, and perform continual successive revolutions in the same; it follows that the Moon, may by the same force of Gravity be made to revolve about the Earth, or any

* *Dem.* Let the time in which the Body performs one revolution be divided into equal parts, in the first of which let the Body describe the right line AB (Fig. 23.): in the second part of time, if not prevented, it would go straight on to c , describing the line Bc equal to AB by the first Law of Nature; the lines SA, SB, Sc being drawn, the triangles SBA, ScB will be equal to each other, their bases AB and Bc being equal and their heights S the same (38. *Elem.* 1). When the Body arrives at B , let the Centripetal force by one single impulse turn it out of the straight line Bc into the line BC : in which let it move on uniformly without receiving a second impulse till it comes to C . Let Cc be drawn parallel to SB meeting BC in C ; then at the end of the second part of time the Body will be found in C , having described the Diagonal of the Parallelogram Nc (Chap. 4. §. 2.). Draw SC , and the triangle SCB will be equal to the triangle ScB , (each having the same base SB and being between the same parallels Cc and SB) and therefore also equal to the triangle SBA . For the same reason, if the Centripetal force acts in the points C, D, E successively, so as to make the Body describe the straight lines CD, DE, EF , &c. in so many equal parts of time, the triangles SCD, SDE, SEF , &c. will be all equal to one another and to the triangle SAB . Consequently equal Areas are described in equal times. Let us then suppose the bases of those triangles, *viz.* AB, BC, CD, DE , &c. diminished in infinitum, and likewise the times in which they are described; then will the Perimeter A, B, C, D, E, F , &c. become a Curve, and any number of those triangles taken together, (or their Areas) will be proportionable to the times in which they are described. *Q. E. D.*

other

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other Planet by the like force about the Sun, if the velocities with which they move are duly adjusted to the forces by which they are acted upon.

WHEN a Body revolves about another in this manner, that force or power by which it is prevented from flying off (as it otherwise would do in a Tangent to the Curve which it describes) is call'd the *Centripetal*; the counter-action of this, by which it endeavours to fly off, the *Centrifugal*; these, by the 3^d Law of Nature being equal to each other, are called by one common name *Central Forces*; that with which the Body is at first projected, or continues its motion from any point, is the *Projectile* force; and the time in which it performs one revolution, the *Periodical* time.

THESE forces properly relating to the motions of the Heavenly Bodies will be more largely treated of in another place.

C H A P. IX.

Of the Communication of Motion.

I. **B**EFORE we proceed to explain the Laws, by which Bodies communicate their motion from one to another, it is very necessary to make a distinction between Motion and Velocity; which ought to be well observ'd and is as follows.

By

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By the Motion of a Body (sometimes called its quantity of motion, sometimes its *Momentum*) is not to be understood the velocity only, with which the Body moves; but the sum of the motion of all its parts taken together: consequently the more matter any Body contains, the greater will be its motion, though its velocity remains the same. Thus, supposing two Bodies, one containing ten times the quantity of matter the other does, moving with equal velocity; the greater Body is said to have ten times the motion or Momentum that the other has: for 'tis evident that a tenth part of the larger has as much as the other whole Body. In short that quality in moving Bodies which Philosophers understand by the term Momentum or motion, is no other than what is vulgarly call'd their *Force*, which every one knows to depend on their quantity of matter as well as their velocity. This is that power a moving Body has to affect another in all actions that arise from its motion, and is therefore a fundamental Principle in Mechanics. See it farther explained in the next Chapter.

II. Now since this Momentum or Force depends equally on the quantity of matter a Body contains, and on the velocity with which it moves; the method to determine how great it is, is to multiply one by the other. Thus suppose two Bodies, the first having twice the quan-

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quantity of matter and thrice the velocity which the other has; any two numbers that are to each other as two to one, will express their quantities of matter (it being only their relative velocities and quantities of matter which we need consider), and any two numbers that are as three to one, their velocities; now multiplying the quantity of matter in the first *viz.* 2 by its velocity 3, the product is 6; and multiplying the quantity of matter in the second by its velocity, *viz.* 1 by 1, the product is one; their relative forces therefore or powers will be as 6 to 1, or the Moment of one is six times greater than that of the other. Again if their quantities of matter had been as 3 to 8 and their velocities as 2 to 3, then would their Moments have been as 6 to 24, that is, as 1 to 4.

THIS being rightly apprehended, what follows concerning the Laws of Communication of Motion by Impulse, and the Mechanical Powers will be easily understood.

The Communication of Motion.

I. In Bodies not Elastic.

III. THOSE Bodies are said to be not *Elastic*, which when they strike against one another do not rebound, but accompany one another after Impact as if they were joyned. This proceeds from their retaining the impression made upon their surfaces after the impressing force ceases to act. For all rebounding

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ing is occasioned by a certain spring in the surfaces of Bodies, whereby those parts, which receive the impressi^on made by the stroke, immediately spring back and throw off the impinging Body; now this being wanting in Bodies void of Elasticity there follows no separation after Impact.

IV. WHEN one Body impinges on another which is at rest, or moving with less velocity the same way, the quantity of the motion or Momentum in both Bodies taken together remains the same after Impact, as before; for by the 3^d Law of Nature, the reaction of one being equal to the action of the other, what one gains the other must lose.

THUS, suppose two Bodies one impinging with 12 degrees of velocity on the other at rest: the quantities of Matter in the Bodies being equal, their Moments and velocities are the same; the sum in both 12; this remains the same after Impact (§. 4.), and is equally divided between them (§. 3.); they have therefore 6 a piece, that is the impinging Body communicates half its velocity and keeps half.

V. WHEN two Bodies impinge on each other by moving contrary ways, the quantity of motion they retain after Impact, is equal to the difference of the motion they had before; for by the 3^d Law of Nature, that which had the least motion, will destroy an equal quantity in the other, after which they will

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will move together with the remainder, that is the difference.

THUS for instance, let there be two equal Bodies moving towards each other, the one with 3 degrees of velocity, the other with 5, the difference of their Moments or velocities will be 2; this remains the same after Impact (§. 5.) and is equally divided between them (§. 3.) they have therefore one a piece: that is, the Body which had 5 degrees of velocity, loses 3 or as much as the other had, communicates half the remainder, and keeps the other half*.

II. In Elastic Bodies.

VI. BODIES perfectly *Elastic* are such as rebound after Impact with a force equal to that with which they impinge upon one another: those parts of their surfaces, that receive the impression, immediately springing back, and throwing off the impinging Bodies with a force equal to that of Impact.

* From these positions it is easy to deduce a Theorem, that shall shew the velocity of Bodies after Impact in all cases whatever. Let there be two Bodies *A* and *B*, the velocity of the first *a* of the other *b*; then (§. 2.) the Moment of *A* will be expressed by *Aa*, and of *B* by *Bb*: therefore the sum of both will be *Aa + Bb*; and *Aa - Bb* will be the difference when they meet. Now these quantities (by §. 4. and 5.) remain the same after Impact; but knowing the quantities of motion and quantities of matter, we have the velocity (which §. 3. is the same in both) by dividing the former by the latter, (as follows from §. 2.) therefore $\frac{Aa+Bb}{A+B}$ or $\frac{Aa-Bb}{A+B}$ will in all cases express the velocity of the Bodies after Impact,

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VII. FROM hence it follows that the action of Elastic Bodies on each other (that of the spring being equal to that of the stroke), is twice as much as the same in Bodies void of Elasticity. Therefore when Elastic Bodies impinge on each other, the one loses and the other gains twice as much motion as if they had not been Elastic; we have therefore an easy way of determining the change of motion in Elastic Bodies, knowing first what it would have been in the same circumstances, had the Bodies been void of Elasticity.

THUS if there be two equal and Elastic Bodies, the one in motion with 12 degrees of velocity impinging on the other at rest, the impinging Body will communicate twice as much velocity as if it had not been Elastic, that is, (by §. 4.) 12 degrees or all it had; consequently it will be at rest, and the other will move on with the whole velocity of the former.

VIII. It sometimes happens that in Bodies not Elastic, the one loses more than half its velocity, in which case supposing them Elastic it loses more than all; that is, the excess of what it loses above what it has, is negative, or in a contrary direction; thus suppose the circumstances of Impact such, that a Body which has but 12 degrees of velocity, loses 16; the overplus 4 is to be taken the contrary way, that is, the Body will rebound with 4 degrees of velocity. *v. g.* Let it be required to determine

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termine the velocity of a Body after Impact against an immoveable object. Let us first suppose the Object and Body both void of Elasticity: 'tis evident the impinging Body would be stopt or lose all its motion, and communicate none; if they are Elastic, it must lose twice as much (by §. 7.) and consequently will rebound with a force equal to that of the stroke.

IX. It is sufficient if only one of the Bodies is Elastic, provided the other be infinitely hard; for then the impression in the Elastic Body will be double of what it would have been, had they both been equally Elastic: and consequently the force with which they rebound will be the same as if the impression had been equally divided between the two Bodies.

X. THERE are no Bodies that we know of, either perfectly Elastic or infinitely hard; the nearer therefore any Bodies approach to perfection of Elasticity, so much the nearer do the Laws they observe in the mutual communication of their motion, approach to those we have laid down.

XI. Sir *Isaac Newton* made trials with several Bodies, and found that the same degree of Elasticity always appeared in the same Bodies, with whatever force they were struck; so that the Elastic power in all the Bodies he made trial upon, exerted it self in one con-

52 *Communication of Motion.* Part I.

stant proportion to the compressing force. He found the celerity with which Balls of Wool bound up very compact, receded from each other, to bear nearly the proportion of 5 to 9 to the celerity wherewith they met; and in Steel he found nearly the same proportion; in Cork the Elasticity was something less; but in Glass much greater; for the celerity with which Balls of that material separated after percussio, he found to bear the proportion of 15 to 16 to the celerity wherewith they met*.

XII. WE have hitherto supposed the direction in which Bodies impinge upon one another to be perpendicular to their surfaces: when it is not so, the force of Impact will be less, by how much the more that direction varies from the perpendicular; for it is manifest that a direct impulse is the greatest of all others that can be given with the same degree of velocity†.

* Newt. Princip. Phil. pag. 21.

† The force of oblique Percussion is to that of direct, as the Sine of the Angle of Incidence to the Radius.

Dem. Let there be a plane as AD (Fig. 24.) against which let a Body impinge in the point D in the direction BD : which line may be supposed to express the force of direct Impulse, and may be resolved into two others (Chap. 4. §. 2.) BC and BA ; the one parallel, the other perpendicular to the Plane; but that force which is exerted in a direction parallel to the Plane can no way affect it; the stroke therefore arises wholly from the other force expressed by the line BA ; but this is to the line BD , as the Sine of the Angle of Incidence ADB to the Radius; from whence the Proposition is clear.

If

Fig. 21. P. 41.

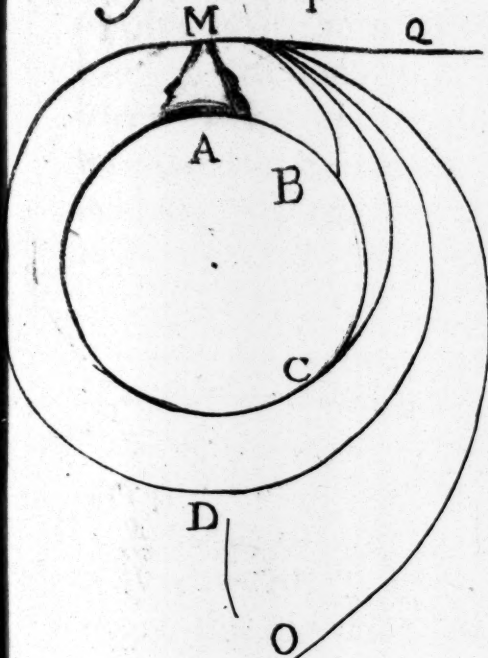
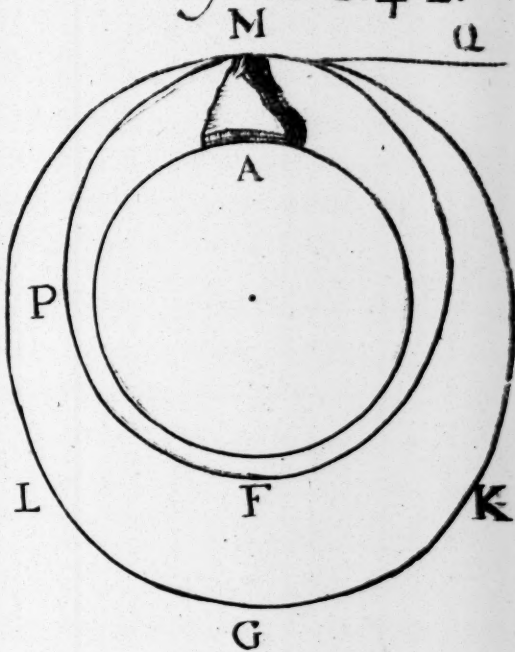


Fig. 22. P. 42.



X
f Fig. 23. P. 44.

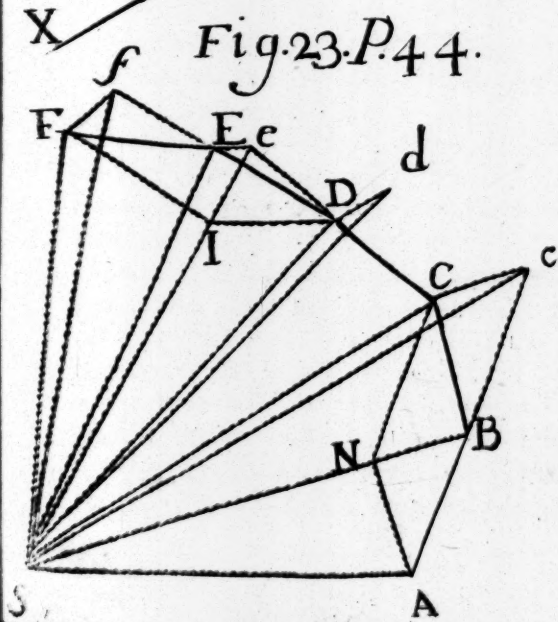
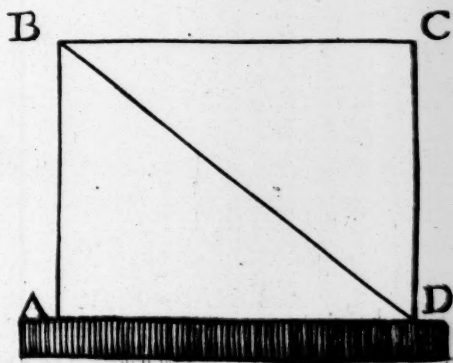


Fig. 24. P. 52.





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XIII. THIS is the case when Bodies impel one another by acting upon their surfaces; but in forces where the surfaces of Bodies are not concerned, as in Attraction &c. we must not consider the relation which the direction of the force has to the surface of the Body to be moved, but to the direction in which it is to be moved by that force. Here the force of action will be less, by how much the more these two directions vary from each other*. My meaning in both cases will be understood from the instance of a Ship under Sail. The force by which the Wind acts upon the Sail, will be less, by how much the more its direction varies from one that is perpendicular to its surface: but the force of the Sail to move the Ship forward, will be less, by how much the more the direction of the Ship's Course varies from that in which she is impelled by the Sail.

If the surface of the Body to be struck is a Curve, then let AD be made a Tangent to D the point of Incidence, and the Demonstration will be the same.

* The force of oblique action is to that of direct, as the Co-Sine of the Angle comprehended between the direction of the force, and that wherein a Body is to be moved thereby, to the Radius.

Dem. Let FD (Fig. 25.) represent a force acting upon a Body as D , and impelling it towards E ; but let DM be the only way in which it is possible for the Body to move; the force FD may be resolved (Chap. 4 §. 2.) into two others FG and FH , or which is equal to it GD ; but 'tis evident that only the force GD impels it towards M . Now, FD being the Radius, GD is the Co-Sine of the Angle FDG comprehended between the two directions FE and GM ; from whence the proposition is clear.

XIV.

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XIV. To this we may add the following Proposition, relating to oblique forces, *viz.* that if a Body is drawn or impelled three different ways at the same time by as many forces acting in different directions; and the quantity of those forces is such that the Body is kept in its place by them: then will the forces be to each other as the several sides of a triangle drawn respectively parallel to the directions in which the forces act*.

* *Dem.* Let the lines AB , AD , AE , (*Fig. 26.*) represent the 3 forces acting upon the Body A in those directions, and by that means keeping it at rest in the point A . Then the forces EA and DA will be equivalent to BA otherwise the Body would be put into motion by them (*contra Hypoth.*) But these forces are also equivalent to AC (*Chap. 4. §. 2*) consequently AC may express the other force, and EC , which is parallel and equal to AD , may express that force: but ACE is a triangle whose sides are all parallel to the given directions, therefore the sides of this triangle will express the relation of the forces by which the Body is kept at rest. $\therefore E. D.$

C H A P.

C H A P. X.

Of the Mechanical Powers.

I. **H**AVING in the foregoing Chapter accounted for the Communication of Motion by Impulse; we proceed next to consider motion as communicated without Impulse; which is done by means of certain Instruments, commonly known by the names of *Mechanical Powers*. The use of these Powers consists chiefly in managing great Weights or performing other Works with a determinate force.

II. THEY are usually reckoned five. *viz.* The Lever, the Wheel and Axis, the Pully, the Screw, and the Wedge; to which some add the Inclined Plane. To these all Machines how complicated soever are reducible.

III. THESE Instruments have been of very ancient use; for we find that *Archimedes*, was well acquainted with the extent of their Power, as may be inferred from that celebrated saying of his, $\Delta\omicron\varsigma\ \pi\epsilon\tilde{\iota}\ \tau\tilde{\omega},\ \kappa\iota\ \tau\tilde{\iota}\omega\ \gamma\tilde{\iota}\omega\ \kappa\iota\tilde{\nu}\eta\sigma\omega$. By which he meant that the greatest imaginable Weight might be moved with the smallest Power.

IV. THAT Body which communicates motion to another, is called the *Power*; that which receives it, the *Weight*.

V.

V. THAT point in a Body which remains at rest, while the Body is turning round, is called the *Center of Motion*. Besides this, there are two other Centers in Bodies, 1. that of *Magnitude*, which is a point, as near as possible, equally distant from all the external parts of the Body; 2. that of *Gravity*, or that about which all the parts of the Body, in whatever situation it is placed, exactly balance each other.

VI. WHEN a Body communicates motion to another, it loses just so much of its own, as it communicates to that other; the action of one being equal to the reaction of the other. See Chapter the last §. 4. and 5.

VII. WHEN two Bodies have such relation to each other (suppose them fixed to different parts of the same Machine) that if one be put into motion, the other will thereby have necessarily such a degree of velocity given it, that their Moments* will be equal; it will then be impossible that one should begin to move without communicating to the other a

* It was proposed (Chapter the last) to give some farther explication of the term *Momentum* in this place, and to shew that the Force or Power any Body has (except such as does not proceed from motion) wholly depends upon it: it being then intended to treat this Subject in the usual way. But the method here made use of renders such explication unnecessary; 'tis sufficient if the Reader understands by it the quantity of motion in a Body, or its quantity of matter multiplied by its velocity, as defined in that Chapter.

quantity of motion equal to its own; 'tis evident therefore from the last Proposition, that if we suppose it to begin to move; in that very instant it must lose all its own motion by communicating it to the other Body: and consequently will remain at rest, communicating none at all. Now the Moments of two Bodies are equal (Chap. 9. §. 2.) when the velocity of the first is to that of the second, as the quantity of matter of the second to that of the first; for if we suppose their quantities of matter as 1 to 3, then by the supposition their velocities are as 3 to 1; and if we multiply the quantity of matter in the first *viz.* 1, by its velocity 3, and that of the other *viz.* 3 by its velocity 1; their products are equal; their Moments are therefore by the Definition (Chap. 9. §. 1. and 2.) equal. They will also be equal, when the spaces the Bodies pass over are in that proportion; for the times they both move in being the same, the spaces will always be as the velocities.

VIII. FROM hence it follows, that in any Machine whether simple or compound, the Power however small may have a Moment equal to that of the Weight; provided the Machine be such, that when it is in motion, the velocity of the Power shall exceed that of the Weight, as much as the Weight is larger than the Power; for then what the Power wants in quantity of matter or weight

H

will

will be made up in velocity; consequently their Moments will be equal by §. the last, and therefore by §. 7. they will exactly balance each other; or be in *Æquilibrio*.

IX. BUT if the Power bears a greater proportion to the Weight, than the velocity of the Weight to that of the Power; it will then have a greater Momentum than the other, and consequently may communicate such a Momentum to it as it will receive, without losing all its own; the remainder therefore, if sufficient to overcome the friction of the Machine, will put it into motion. We proceed now to treat of each Mechanical Power in its order, and

I. Of the LEVER.

X. THE Lever is a right line (or bar whose weight in Theory is not considered) moveable on a Center, which is called its *Fulcrum*, or *fixed Point*.

XI. THE *Æquilibrium* in this Machine is, when the distance of the Power from the fixed point is to that of the Weight from the same, as the quantity of matter in the Weight to that in the Power.

FOR supposing the Lever placed on its Fulcrum with the Weight to be raised at one end, and the Power applied to the other; 'tis evident the farther the Power is placed from the Fulcrum or center of motion, the larger will be its sweep when the Machine is put in motion; that is, it will move over so much

much more space in the same time than the Weight to be raised: now if it is placed just so much farther from the Fulcrum, as it is less than the Weight, it will move just so much faster; their Moments therefore will be equal (§. 7.) and consequently the Power and Weight will exactly balance each other, or be in *Æquilibrium**. And if the Power is sufficiently augmented to overcome the friction of the Machine, it will put it in motion.

THE Lever is of 3 kinds. 1. When the fixed point is between the Weight and the Power, as in the last case. 2. When the Weight is between the fixed point and the Power. 3. When the Power is between the fixed point and the Weight.

IN all which cases the *Æquilibrium* will be, when their distances from the fixed point are such, that their velocities shall be inversely as their quantities of matter; for then by §. 7. being at rest, neither of them will communicate any motion to the other.

* Geometrically thus. Let *AB* (*Fig. 27.*) represent the Lever, *F* the Fulcrum, *W* the Weight, *P* the Power, the one suspended at the extremity of the Lever *A*, the other at *B*, and let *BF* be to *FA* as *W* to *P*; then while the Lever moves from the situation *AB* into that of *CD*, the point *B* which sustains the Power will move as much farther than *A* which sustains the Weight (and consequently as much faster since they perform their motions in the same time) as the arch *BD* is longer than *AC*; that is, the triangles *BFD* and *AFC* being similar, as the arm *BF* is longer than *AF*, which (*ex Hypoth.*) is as much as the Weight exceeds the Power, they will therefore (§. 7.) be in *Æquilibrium*. Q. E. D.

THE common Scales may be considered as a Lever of the first kind, where the Weight and Power are applied at equal distances from the fixed point.

THE Steelyard is also a Lever of the first kind, whose arms are unequal.

THE difference between the use of the Scales and the Steelyard consists in this; that as in one you make use of a larger Power (or more Weights) to estimate the weight of an heavier Body; in the other you use the same Power, but give it a greater velocity with respect to that of the Weight by applying it farther from the fixed point, which by §. 7. will have the same effect.

II. The WHEEL and Axis.

XII. THIS Machine is a Wheel, that turns round together with its Axis; the Power in this is applied to the Circumference of the Wheel, and the Weight drawn up by means of a Rope wound about the Axis.

XIII. IN this there will be an *Æquilibrium*, when the Weight is to the Power, as the Diameter of the Wheel to the Diameter of the Axis.

'TIS evident, the velocity of the Power will exceed the velocity of the Weight, as much as the Circumference of the Wheel exceeds that of its Axis; because the spaces they pass over in one revolution will be as those Circumferences; that is, as much as the Diameter
of

of one exceeds that of the other, (the Circumferences of Circles being as their Diameters;) what therefore in this case the Power wants in weight will be made up in velocity, from whence (§. 7.) there will be an *Æquilibrium* *.

THE use of this Machine is to raise Weights to greater heights than the Lever can do, because the Wheel is capable of being turned several times round, which the Lever is not; and also to communicate motion from one part of a Machine to another; accordingly there are few compound Machines without it.

III. The PULLEY.

XIV. A Pulley is an Instrument composed of one or more Wheels moveable on their Axes.

XV. A simple Pulley, if its Axis is fixed, is of no other use, than to alter the direction of the Power; for the Power and Weight will both move through an equal space in the same time. But in a Pulley not fixed, as in *Fig. 29.* where the Rope runs under it, or in a combination of Pullies as in *Fig. 30.* the *Æquilibrium* will be, when the

* Geometrically thus. Let *AB* (*Fig. 28.*) be the Diameter of the Wheel, *DE* that of the Axis, *W* the Weight, and *P* the Power; when the Wheel begins to move, the point *B* and *D* will describe similar Arches about the Center *C*, in the same manner the point *A* and *B* in the Lever were shewn to do about the fixed point *F* (*Fig. 27*) that is the point *B* will move as much faster than *D*, as *CB* is longer than *CD* or *AB* than *DE*, the motion therefore of *P* (§. 7.) will be equal to that of *W*. From whence the Proposition is clear.

Power is to the Weight, as one to the number of Ropes, that pass between the upper and lower Pullies.

SUPPOSE one end of the Rope fixed in B (*Fig. 29.*) the other supported by the Power P, it is evident, that in order to raise the Weight W one foot, the Power must rise two, for both Ropes *viz.* BC and CP, will be shortened a foot apiece, whence the space run over by the Power, will be double to that of the Weight; if therefore the Power is to the Weight as 1 to 2, their Moments will be equal: for the same reason if there be 4 Ropes passing from the upper to the lower Pullies as in *Fig. 30.* the velocity of the Power will be quadruple to that of the Weight, or as 4 to 1. &c. In all cases therefore when the Power is to the Weight, as one to the number of Ropes passing from the upper to the lower Pullies, (§. 7.) there will be an *Æquilibrium*.

XVI. IF the Pullies be disposed as in Figure the 31st, each having its own particular Rope, the action of the Power will be very much increased; for here every Pulley doubles it, wherefore the Power is 4 times greater with 2 Pullies, 8 times with 3, 16 times with 4 &c. For it is evident from the consideration of the Figure, the first will move half as fast as the Power, the second half as fast as that, and so on; wherefore (§. 7.) the Power is doubled by each Pulley.

THE

Plate VII. Pag. 62.

Fig. 25. P. 53.

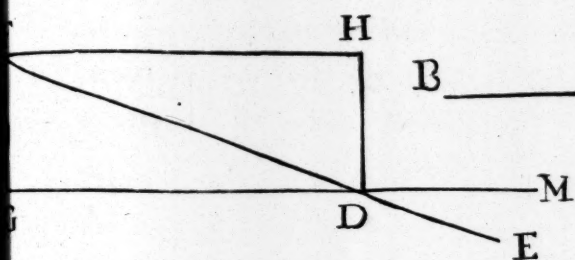


Fig. 26. P. 54.

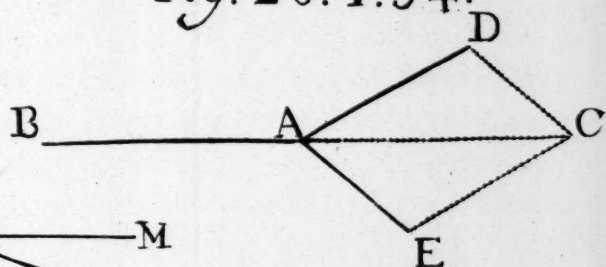


Fig. 27. P. 59.

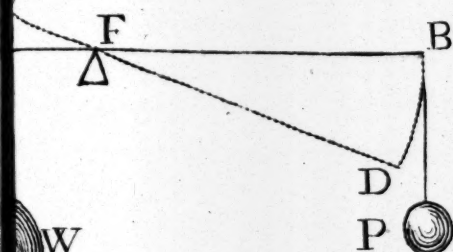


Fig. 28. P. 61.

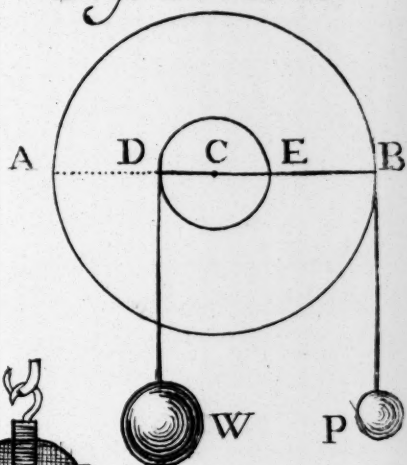
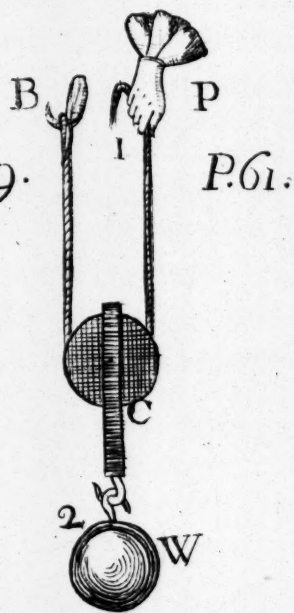
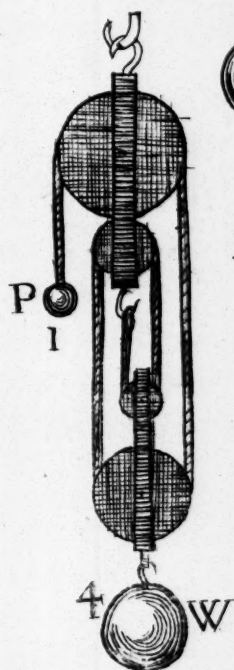


Fig. 29.



P. 61.

Fig. 30.





THE use of the Pulley is nearly the same with that of the Wheel and Axis, but it is more portable and easier to be fixed up.

IV. The SCREW.

XVII. IN this Machine the *Æquilibrium* will be, when the Power is to the Weight, as the distance between any two contiguous threads or spirals in the Screw, to the way described by the Power in one whole revolution. It is manifest from the form of the Machine (*Fig. 32.*) that in one revolution of the Screw, the Weight will be moved through a space equal to the distance of two contiguous threads, and that the Power will run through a space equal to the compass it takes in one revolution, therefore (§. 7.) if the Weight exceeds the Power in this proportion, there will be an *Æquilibrium*.

THIS Machine is of great force, and very useful in retaining Bodies in a compressed state, because it will not run back, as the three foregoing will when the Power is removed. This arises from the great friction of those parts in the Screw, which during its motion slide upon those, that are at rest.

V. The WEDGE.

XVIII. THIS Instrument is formed by two equal rectangles joined at their lower bases, and separated at their upper ones, by a third; which is called the *Back* of the *Wedge*; the other two, its *Sides*.

XIX.

XIX. IN the foregoing Mechanical Powers we have all along considered the Weight, as moved in the same direction with that in which it is acted upon by the Machine, as is commonly the case; but in this, the Weight is generally applied in such a manner as to be made to move in a direction different from that in which it is protruded by the Wedge; hence it is, that Mathematicians have differed in their determination of the Power of this Machine, some considering the Weight as moved by it in one direction and some in another. Nay there are some (I speak of late Writers) that have differed from Truth it self. We will therefore lay down the several Proportions they have given us for the determining the Power of this Machine and examine them one by one.

1. It is demonstrated by some, that the Power will be equivalent to the resistance of the Weight, when it bears such proportion to it, as the breadth of the Back of the Wedge, does to the sum of its Sides; or, which is the same thing, as half that breadth to one of its Sides. 2. Others make it somewhat larger, and demonstrate that it ought to be as half the breadth of the back to the perpendicular height of the Wedge. 3. Some are of opinion, that there will not be an *Æquilibrium* in this Machine, unless the Power is to the Weight, as the whole breadth of the Back to the perpendicular height. *Wallis, Keil &c.* 4.

Grave.

Gravesande in his *Elements* (L. I. Ch. 13.) gives us the same proportion with the last; and in his *Scholium de ligno findendo*, tells us, that when the parts of the wood are separated no farther than the Wedge is driven in, the *Æquilibrium* will be, when the Power is to the Resistance, as half the breadth of the Back of the Wedge to one of its Sides.

THOSE who lay down the first Proportion for determining the Power of this Machine, suppose the parts, which are separated from each other thereby, to recede from their first situation in directions perpendicular to the sides of the Wedge. Thus let ACB (*Fig. 33.*) represent a Wedge; P, P, two Bodies to be separated by it, the one to be moved towards I, the other towards F, in the directions CI and CF perpendicular to AC and CB; then 'tis evident that when the Wedge is driven in to the situation MNO, the two Bodies will be moved to Q and Q; that is, one will have passed through the space CK the other through CL, but these spaces being equal, their velocities are the same as if they had both passed over one of them. *v.g.* CL, or which is equal to it DG (drawn perpendicular to CB); therefore the Power which we suppose applied at D moves through DC, while the obstacle moves through DG, consequently (§. 7.) when the Power is to the Weight as DG to DC,

I

that

that is, as DB to CB^* , or half the Back of the Wedge to one of its Sides, they will be in *Æquilibrium*. This proportion therefore, when the parts of the Weight are moved by the Wedge in the directions CI and CF , may be admitted as true.

2. THE second proportion is also true, supposing the Bodies P, P , to recede from each other in the directions CN, CM , parallel to AB the Back of the Wedge; for when the Wedge is driven in between them, to the situation MNO , the Bodies will have moved through a space as CN , or which is equal to it DB , half the Back of the Wedge, and the Power through a space equal to its height as before; consequently (§. 7.) in this case, the *Æquilibrium* will be, when the Power is to the Weight, as half the Back of the Wedge to its height †.

* For (8. *Elem.* 6.) the triangles DCG and DCB are similar, and consequently $DG : DC :: DB : BC$.

† The same may be otherwise demonstrated from Section 14. Chapter 9 thus. Let there be a Body as L (*Fig.* 34.) drawn against the Wedge ABC by the Weight W , in the direction LF ; parallel to the back of the Wedge AB ; but prevented from sliding down towards C by a Plane (whose upper surface we may suppose represented by EF) lying under it. I say, the Power will be to the Weight, when they are in *Æquilibrium*, as DB to DC .

Dem. The Body L is here acted upon in three directions, *viz.* by the force of the Weight W in the direction LF , by the two Planes CB and EF , in the directions LG and LI , perpendicular to their surfaces; let GE be drawn parallel to LI , then will the triangle LGE have all its sides respectively parallel to those directions; consequently (Chap. 9. §. 14.) if we suppose LE to express

3. THOSE, who imagine there will not be an *Æquilibrium*, unless the Power be to the Weight, as the whole breadth of the Back of the Wedge to its height, suppose as in the last case, that the Bodies to be separated, recede from each other in directions parallel to the Back of the Wedge; and endeavour to support their opinion by the following Argument: *viz.* that, when the Wedge is driven in to the situation *MNO* (*Fig. 33.*) as before, each part of the Weight having moved through a space equal to half the Back of the Wedge, the whole Weight has therefore moved through twice so much, or a space equal to the whole Back: as much as to say, the whole has moved farther than its parts; which is absurd.

express the force of the Weight *W*, *GL* will represent the pressure of the Body *L* against the Wedge; and if that is resolved into *GE* and *GH* the one perpendicular to the direction of the Power, the other parallel and contrary to it; the last, *viz.* *GE*, will express the whole force of the Weight to resist the motion of the Power; but *GE* is to *EL*, as *DB* to *DC* (for the triangle *EGL* and *DBC* are similar, the sides of one being *ex Construct.* respectively perpendicular to those in the other; *v. g.* *LG* to *CB*, *EL* to *DC* and *GE* to *DB*); consequently the Power is to the Weight, when they balance each other, as half the breadth of the Back of the Wedge to its height. *Q. E. D.*

Corol. Suppose the Body *L* had been drawn against the Wedge in the direction *GL* perpendicular to its surface, and to be moved by the Wedge in the contrary direction towards *G*, as in the first case; then if *GL* expresses the force with which it is drawn towards the Wedge, *GE* will be that with which it resists the Power; but *GE* is to *GL* as *DB* to *BC*, the triangles *EGL* and *DBC* being similar; consequently in this case, the Power will be to the Weight, as half the breadth of the Back of the Wedge to one of its sides; as was before demonstrated.

4. THIS is *Gravesande's* mistake in his *Elements*, the same he has also made in his *Scholium de ligno findendo*, and thereby determined the Power in both places to be twice as big, as it ought to be. If he had proceeded in the following manner, his Argument would have been easier, as well as the Conclusion juster. Suppose the Wedge ABC driven into the Wood QLQ, (as represented *Fig. 35.*) which is split no farther than the point of the Wedge, or however no farther than is just sufficient to give it room to move, I say that in this situation of the Wedge, the Power is to the Weight, as one fourth part of the Back of the Wedge to one of its Sides. For it is evident, that when the upper ends of the Wood, which press against the Wedge in the points G, H, are put into motion by the Wedge, they will move in the directions HI and GF, perpendicular to the sides of the Wedge, because they turn as it were upon a joynt at L, which we suppose contiguous to C: again, since only the upper ends of the Wood are put into motion, and not the lower ones, which remain at L; 'tis evident that the motion of each piece (supposing their thickness the same from end to end, and their substance uniform) will be but half, what it otherwise would have been. Now were all the parts of the Wood to have the same degree of velocity, the Power would be to the Weight, as in the first case, *viz.*

as

Fig. 31. P. 62.

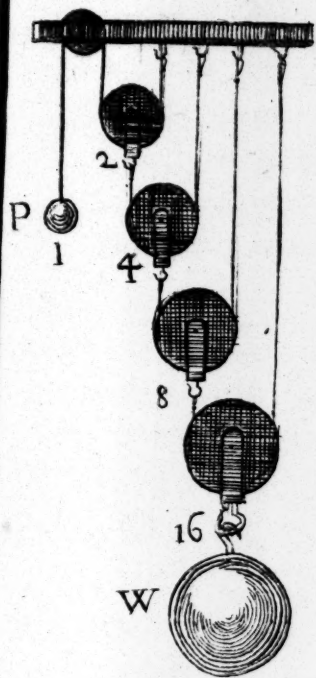


Fig. 32. P. 63.

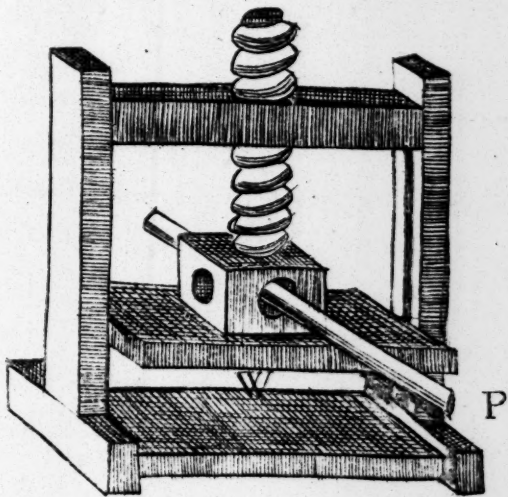


Fig. 33. P. 65.

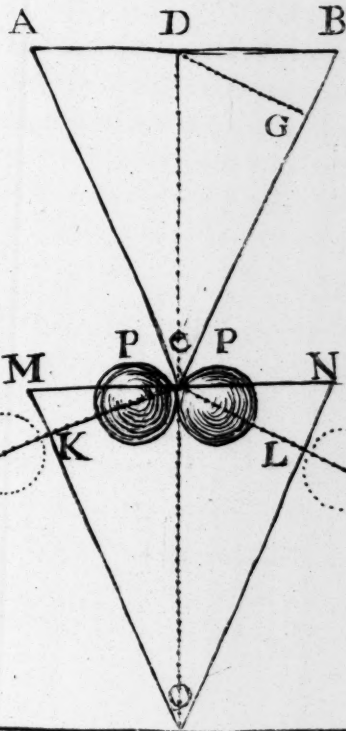
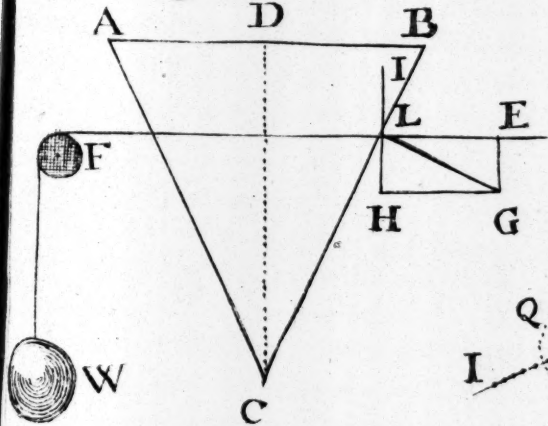


Fig. 34. P. 66.





as DB to BC (*Fig. 33.*); therefore in this case, it is as half DB to BC, or as one fourth part of the Back of the Wedge to one of its Sides. Which was to be proved.

XX. THE form of the *Inclined Plane* being no other than that of half a Wedge, as is manifest from the representation of it (*Fig. 36.*) it follows that what has been demonstrated of the one, may be applied to the other, and the properties of both will be the same. For instance, if the Weight *W* is to be raised up the Plane *CB*, by the Power *P*, in a direction parallel to the Plane; instead of that, we may suppose the Weight prevented from running off the Plane by the String *WB*, and the inclined Plane driven under it like a Wedge in the direction *DC*: then will the Weight rise towards *G* in a direction perpendicular to *CB*, for we must always suppose the String *CB* parallel to the Plane, as it would have been, if the Weight had been drawn up by it; then will the action of the Plane upon the Weight be similar to that of the Wedge in the first case; and consequently the Power will bear such proportion to the Weight, as DB to BC; that is, as the height of the Plane to its length. Again, suppose the Weight was to have been drawn up the Plane by a String in the direction *WF* parallel to the base of the Inclined Plane *CB*; then if the Plane be driven under the Weight as before, it must rise in a direction

direction perpendicular to CD, that is parallel to DB: then the case will be analogous to the 2^d of the Wedge; consequently the Power will be to the resistance of the Weight, when there is an *Æ*quilibrium, in the proportion of DB to DC, as there demonstrated.

XXI. THESE are the Powers or Machines, which under different forms, constitute all others how complicated soever; and as the *Æ*quilibrium in any one of these is, when the Power and Weight are inversely as their velocities; so in a Machine however compounded, the Power and the Weight will exactly balance each other, when they are in this proportion; for by §. 7. their Moments will then be equal, and the Machine, if at rest will continue in that state; and if put into motion by an external force, will gradually lose it, when that force ceases to act; on account of the unavoidable friction of the Machine, and the resistance of the Air, which it must necessarily meet with, unless its motion could be performed in a perfect Vacuum. From hence we see the impossibility of contriving an Engine, whose motion should be *perpetual*, that is, such as does not owe its continuance to the application of some external force; a Problem that has given birth to an almost infinite number of Schemes and Contrivances. For unless some method could be found out of gaining a force, by the artful disposition
and

and combination of the Mechanical Powers, equivalent to that which is continually destroyed by friction, and the resistance of the Air, the motion which was at first given to the Machine must at length be necessarily lost. But we see, that those Instruments are only different means, whereby one Body communicates its motion to another; and not designed to produce a force which had no existence before. A given force may be disposed between the Power and the Weight an infinite number of ways; but can never be augmented by any Mechanism whatever: so much as we place in the Power will always be lost to the Weight, and what we attribute to the Weight will never be found in the Power. 'Tis for want of a due consideration of this, that so many Mechanical Designs have proved abortive, so many Engines unequal to the performance for which they were designed, and so many impossibilities attempted.

“ If it were possible, says Bp. *Wilkins*, to
“ contrive such an invention, whereby any
“ conceivable Weight may be moved by any
“ conceivable Power, both with equal velocity
“ (as it is in those things which are immediately
“ stirred by the hand, without the help of any
“ other instrument) the works of Nature
“ would be then too much subjected to the
“ power of Art; and Men might be thereby
“ encouraged (with the Builders of *Babel*, or
“ the

“ the rebel Giants) to such bold designs, as
 “ would not become a created Being. And
 “ therefore the Wisdom of Providence has so
 “ confined these human Arts, that what an
 “ invention hath in the *strength* of its motion,
 “ is abated in the *slowness* of it; and what it
 “ has in the extraordinary *quickness* of its mo-
 “ tion, must be allowed for in the great *strength*
 “ requisite in the Power, which is to move
 “ it*.

* Wilkins's Mathem. Magick, p. 104.



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